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QCD Sum Rules and Applications to Nuclear Physics

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Abstract

Applications of QCD sum-rule methods to the physics of nuclei are reviewed, with an emphasis on calculations of baryon self-energies in infinite nuclear matter. The sum-rule approach relates spectral properties of hadrons propagating in the finite-density medium, such as optical potentials for quasinucleons, to matrix elements of QCD composite operators (condensates). The vacuum formalism for QCD sum rules is generalized to finite density, and the strategy and implementation of the approach is discussed. Predictions for baryon self-energies are compared to those suggested by relativistic nuclear physics phenomenology. Sum rules for vector mesons in dense nuclear matter are also considered.

Keywords: Quantum chromodynamics, QCD sum rules, hadronic properties in nuclei, finite-density condensates, Dirac phenomenology, quantum hadrodynamics

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Contents

I. INTRODUCTION

A. Quantum Chromodynamics and Nuclear Physics

There is little disagreement today that quantum chromodynamics (QCD) is the correct theory underlying strong-interaction physics. Thus, the physics of nuclei is, in essence, an exercise in applied QCD. Indeed, from a fundamental perspective, *the* central problem in theoretical nuclear physics is to develop connections between observed nuclear phenomena and the interactions and symmetries of the underlying quark and gluon degrees of freedom.

On the other hand, knowledge of the fundamental underlying theory has had very little impact, to date, on the study of low- and medium-energy nuclear phenomena. Two general difficulties hinder the application of quantum chromodynamics to nuclear physics. The first is that, in this energy regime, the strong interaction is, in fact, strong; straightforward perturbation theory in the effective QCD coupling constant fails. Therefore we need an alternative expansion scheme or an approach to directly approximate QCD nonperturbatively.

The second difficulty is the mismatch of energy scales between hadronic and nuclear physics. Characteristic QCD scales for light-quark hadrons, which are the building blocks of nuclei, are hundreds of MeV up to several GeV. The dynamics of nuclei, in contrast, is a delicate and subtle enterprise involving physics at much smaller scales and featuring sensitive cancellations. Typical nuclear observables are on the scale of a few MeV or perhaps tens of MeV. Consider, for example, the phenomenon of nuclear matter saturation. The binding energy per nucleon is approximately 16 MeV [1], which is less than 2% of the nucleon's mass. An error of a few percent in the binding could easily lead to errors at the level of nuclear physics scales, which would render any calculation useless. Thus, an accurate description of nuclear matter *saturation* or comparable properties directly from QCD must entail very precise calculations.

How, then, might one proceed to relate QCD and nuclear phenomena? An *indirect* approach is to focus on the implications and constraints of QCD symmetries by developing effective field theories (EFT's) for nuclei. The prototype EFT for strong-interaction physics is chiral perturbation theory (ChPT), which provides a systematic expansion in energy for low-energy scattering processes. The degrees of freedom are the Goldstone bosons of spontaneously broken chiral symmetry (pions, *etc.*) and, when appropriate, nucleons. This approach builds in constraints due to chiral symmetry without any additional constraints on the dynamics or *ad hoc* model assumptions. Physics beyond chiral symmetry is incorporated through constants in the low-energy Lagrangian, which are usually determined from experiment, although in principle they could be determined directly from QCD. Because additional constants are needed at each stage in the energy expansion, ChPT is predictive only at sufficiently low energies, where the number of parameters introduced does not overwhelm the data to be described. The prospects for extending ChPT in a useful way to calculations at finite density are unclear at present, although some progress has been made on the few-body problem [2].

Alternatively, we can pursue a more *direct* approach by focusing on the properties of hadrons rather than nuclear matter saturation. If we can successfully describe hadronic resonances in the vacuum using QCD, we might be able to extend this description to hadronic properties at finite density. The most complete approach to hadronic physics at zero density

uses Monte Carlo simulations of QCD on a discrete (Euclidean) space-time lattice. Significant and steady progress has been made on such calculations in recent years. (For a review of the current state of the art, see Ref. [3].)

The prospects for extending these lattice calculations to finite density are unclear; there are certainly formidable obstacles. One major difficulty is that the functional determinant that arises with a nonzero chemical potential is not positive definite, and hence standard Monte Carlo techniques cannot be applied in a straightforward manner [4]. In addition, realistic nuclear matter calculations may require significantly larger lattices than are required for single hadrons; there must be enough room for a sufficiently large number of nucleons. Even with significant progress on these problems, useful lattice calculations at finite density will not be available in the near future.

To make progress, we turn instead to QCD sum rules. The QCD sum-rule approach has proven to be a useful way of extracting qualitative and quantitative information about hadronic physics (masses and coupling constants) from QCD inputs. The method was introduced by Shifman, Vainshtein, and Zakharov in the late 1970's [5] and applied to describe mesonic properties. An important extension was made by Ioffe, who showed how the technique could be used to describe baryons [6] (see also [7]). There are several detailed reviews of the subject [8–11].

In this review, we will discuss how QCD sum-rule methods can be generalized to describe hadrons propagating in nuclear matter. The pioneering attempts to apply sum rules to finite-density systems were made by Drukarev and Levin [12]. Subsequently, a variety [13–27] of approaches were developed, differing in focus and technical detail. Here we will emphasize the treatment of baryons in infinite nuclear matter, and we will closely follow the philosophy and technical approach of Refs. [14,18,20,25] for nucleons and Refs. [22,27] for hyperons. The properties of vector mesons in the nuclear medium will be discussed following the treatment in Refs. [19,24]. We will also comment on alternative formulations of the problem.

B. QCD Sum Rules

QCD sum rules focus on momentum-space correlation functions (also called correlators) of local composite operators [for example, see Eq. (2.13)]. Each composite operator is constructed using quark and/or gluon fields so as to carry the quantum numbers of the hadron we wish to study; we will refer to such operators as interpolating fields. Correlation functions of interpolating fields are also the basic objects of lattice investigations (but in Euclidean coordinate space). But while the lattice correlators are calculated from first principles in quantum chromodynamics with a precision that can be systematically improved, sum-rule calculations rely on some phenomenological input and have limited (but generally understood) accuracy [10].

The basic idea of QCD sum rules is to match a QCD description of an appropriate correlation function with a phenomenological one. The underlying concept is "duality," which establishes a correspondence between a description in terms of physical (hadronic) degrees of freedom and one based on the underlying quark and gluon degrees of freedom. A generic QCD sum-rule calculation consists of three main parts: 1) an approximate description of the correlation function in terms of QCD degrees of freedom via an *operator product expan-*

sion (OPE), 2) a description of the same correlator in terms of physical intermediate states through a Lehmann representation (*i.e.* dispersion relation) [28] that incorporates a simple ansatz for the spectral density, and 3) a procedure for matching these two descriptions and extracting the parameters of the spectral function that characterize the hadronic state of interest.

The concept of an operator product expansion was introduced by Wilson in the 1960's [29]; the OPE is reviewed in Ref. [30]. As applied in QCD sum rules, the OPE expresses a correlator of interpolating fields as a sum of c-number coefficients (called Wilson coefficients) times expectation values of composite local operators constructed from quark and gluon fields. These expectation values are referred to as *condensates*.

The essence of the OPE is the separation of short-distance and long-distance physics, or, in momentum space, large and small (spacelike) momenta. In QCD this separation, to a large extent, corresponds to a separation between perturbative physics (the coefficients) and nonperturbative physics (the condensates). This suggests that the coefficients can be calculated from QCD via perturbation theory [5], while the nonperturbative physics is isolated in the condensates. In principle, the condensates are calculable directly from QCD (for example, using lattice simulations), but in practice they are usually determined phenomenologically from one set of sum rules and applied to others. The sum rules are predictive because a relatively small number of condensates dominate the descriptions of many observables [10].

The possibility of matching the OPE-based description with the phenomenological description is often viewed by nonpractitioners with skepticism; it seems like magic. The OPE is essentially a short-distance expansion, and at any finite order it can only describe the correlation function accurately at sufficiently large spacelike momenta. On the other hand, we wish to learn about the low-lying excitations in the spectral function, which one expects to dominate the correlation function only at nearby timelike momenta or the low-momentum spacelike regime. How, then, is there any hope of matching the two? The secret is that the phenomenological spectral density (*i.e.*, the sum over physical states) can be smeared in energy to a significant degree while preserving basic information on well-defined low-energy excitations (hadrons). On the other hand, the smearing corresponds to short times so that the dual description of the spectral density in terms of the quark and gluon OPE is adequate. The conversion from a momentum-space correlator to a smeared spectral function is conventionally achieved by applying a Borel transform [5] (see Sec. II E).

As will be discussed in the course of this review, there are definite limitations of the sum-rule approach, and one must take care in applying sum rules to new domains, as we do here. We wish to warn the reader at the outset that it is rather easy to get totally incorrect results when using (or, rather, abusing) QCD sum rules. The basic dangers are that the phenomenological and OPE descriptions at the level of approximation used for each simply may not match well enough to give reliable information or that some new feature of the physics is omitted. If, however, one nevertheless proceeds to extract phenomenological information, this information will likely be spurious. The need for internal and external consistency checks cannot be overstressed.

Keeping this in mind, the extension of the sum rules to finite density is, in principle, straightforward: simply take expectation values in a finite-density ground state rather than in the vacuum. Consequently, we study the propagation of hadrons in infinite nuclear matter. In nature, of course, infinite nuclear matter does not exist. Why then do we study

this somewhat unphysical system rather than real finite nuclei? In fact, infinite nuclear matter is in many ways a more general and fundamental problem. After all, the nuclear interaction saturates, and the centers of medium and large nuclei all look roughly like nuclear matter. By working with an infinite system, we isolate the physics that is related to QCD dynamics and not to details of the many-body physics of finite systems. Furthermore, we can exploit the similarities to the vacuum formulation (including, for example, translational invariance). The vacuum sum rules can be recovered in the zero-density limit, and we can take ratios to divide out certain systematic errors.

The price we pay is that we do not make direct predictions of experimental observables. To do so one needs to make model-dependent assumptions, such as local-density approximations, to relate calculations of nuclear matter properties to experimental observables of finite nuclei. Thus we are ultimately forced to compare with phenomenology rather than with experimental data.

One has to confront, in addition, a basic limitation of the sum-rule approach; namely, the spectral information that can be extracted is necessarily approximate and cannot be systematically improved. For example, the extraction of hadronic masses with a reliability of better than $\sim$100 MeV is typically not possible [8–10]. Thus the approach is not sufficiently precise to make detailed calculations of the nuclear matter ground state at the scale of a few MeV, the natural scale for much of low-energy nuclear physics. However, another limitation of the approach, the fact that it is not a self-consistent dynamical theory, turns out, in this context, to be a virtue. In the sum-rule approach, one does not dynamically solve for the ground state of the theory. Rather, one *characterizes* the ground state: the QCD description of the ground state is characterized by various expectation values of composite operators. Thus the approach need not be precise enough to calculate the ground state reliably; as long as one has a reasonable way to extract these condensates phenomenologically, one can proceed to study the qualitative features of excitations without actually solving the ground-state problem.

Although one can avoid solving for the nuclear matter ground state, the approximate nature of the sum-rule approach means that one must use extreme care if using the method to extract quantities that are small on the scale of hadronic physics. Thus, the use of the technique to describe fine details of low-energy nuclear physics must depend on the construction of reliable sum rules for nuclear properties that are much more accurate (in absolute terms) than the sum rules have proven to be at the hadronic level. Whether or not this will prove to be possible remains an open question, but we view any results for quantities at this scale with some skepticism. Indeed, the question may be posed as to whether there are *any* properties in low- or intermediate-energy nuclear physics whose natural scale is sufficiently large as to make the sum-rule approach reasonable.

C. Dirac Phenomenology

If we go beyond direct experimental observables and turn to nuclear phenomenology, we *can* find quantities in low- and intermediate-energy nuclear physics whose natural scale is several hundred MeV. The quantities we have in mind are the Lorentz scalar and vector parts of the optical potential for a baryon propagating in nuclear matter. In the Dirac

phenomenology of nucleon-nucleus scattering [31–40], the Lorentz scalar piece of the real part of the optical potential for the nucleon is typically several hundred MeV attractive while the (time component of the) Lorentz vector piece is typically several hundred MeV repulsive. We propose that QCD sum rules are a natural and reasonable way to study these quantities, and we focus our review on this goal.

The Dirac description of nucleons propagating in nuclear matter appears to be quite different from the conventional low-energy nuclear physics approach, in which the (central) optical potential is a few tens of MeV. The latter nonrelativistic approach has been quite successful in describing cross sections in proton-nucleus scattering. Dirac phenomenology describes cross sections equally well; the reason is that only the sum of the scalar and vector potentials contributes to the cross section, and, because of large cancellations between the scalar and vector optical potentials, the sum is only a few tens of MeV. Furthermore, most of the phenomenological energy dependence of the nonrelativistic central potential at low energies arises simply from the nonrelativistic reduction of the Dirac potentials, even when the latter are energy independent [39].

The most compelling support for the Dirac approach comes from studies of spin observables such as the analyzing power and the spin rotation. These observables depend on the *difference* between the scalar and vector potentials, and, given the opposite sign, this difference is many hundreds of MeV. The phenomenological evidence in favor of this description is based on direct fits of optical potentials to describe simultaneously the spin observables and cross sections [31–33,38]. Moreover, a relativistic impulse approximation (RIA) gives optical potentials that are qualitatively similar to the empirical fits [34–37]. Further, in relativistic structure calculations, there is evidence that keeping track separately of scalar and vector parts leads to phenomenologically desirable density dependences and nonlocalities [40].

While the Dirac approach has been quite successful phenomenologically, it has also been quite controversial [41–48]. We believe that much of this controversy is misplaced, involving issues of how Dirac phenomenology is to be interpreted rather than the fundamental underlying issues of physics (see Sec. VI C). However, in light of this controversy, it is important to ask whether there is any evidence from QCD in favor of the qualitative picture of Dirac phenomenology. From a phenomenological perspective, the question is not whether the nucleon should be thought of as a point Dirac particle; it is clear that the composite structure of the nucleon plays an essential role in the dynamics. The key question, however, is whether the optical potential for the nucleon has large and canceling pieces that transform as scalars and vectors under Lorentz transformations.

If QCD sum rules can be implemented so as to separately describe the scalar and vector parts of the optical potential, then we can test whether the scalar and vector parts of the optical potential have the sign and scale suggested by Dirac phenomenology.

D. The Chiral Condensate

If the large-scale changes at nuclear matter densities implied by Dirac phenomenology are to be reconciled with QCD through sum rules, we must expect to find correspondingly large changes in the QCD description. Is this plausible? An important clue comes from the chiral condensate, which is the expectation value of the operator $\overline{q}q$, where q is an up or down

quark field. The chiral condensate is often taken as the order parameter for spontaneous chiral symmetry breaking. As shown by Ioffe [6], this condensate plays a central role in the QCD sum-rule determination of the nucleon mass. This role is consistent with chiral models of the nucleon and is plausible from rather general considerations; 't Hooft's [49] anomaly matching conditions require massless nucleons (for massless quarks) if chiral symmetry is not broken. Thus, it comes as no surprise that the amount of chiral symmetry breaking (which is measured by the chiral condensate) is correlated with the nucleon's mass.

So our question becomes: Is the chiral condensate in nuclear matter very different from its free space value? In fact, the chiral condensate in medium can be estimated in a reasonably reliable way (for an overview of the current state of affairs, see the review by Birse [50]). The magnitude of the chiral condensate in the interior of nuclei is reduced from its vacuum value by $\sim$ 30–40% [51] (see Sec. III C). To the extent that the chiral condensate controls the nucleon mass, one might naively expect the mass of a nucleon in the nuclear medium to be reduced by $\sim$ 30–40%. This may seem absurd; after all, the empirical binding energy per nucleon in nuclear matter is approximately 16 MeV [1], while $\sim$ 30–40% of the nucleon mass is several hundred MeV.

In fact, however, in addition to a shift in the nucleon mass (which is a Lorentz scalar), the in-medium nucleon energy can depend on a self-energy that transforms as a Lorentz four-vector. For qualitative consistency with experiment and Dirac phenomenology, one needs to find that the vector contribution is repulsive in nuclear matter and largely cancels the scalar attraction. As we will show, under certain assumptions about condensates, QCD sum rules predict this qualitative behavior for the optical potential [18,20,25].

E. Other Hadrons in Medium

Of course, nucleons are not the only hadrons. The same methods that are used to describe nucleon self-energies in medium can be directly adapted to describe optical potentials (self-energies) of other baryons in nuclear matter. Again, we might expect that large changes in the chiral condensate (along with changes in other condensates) will imply large effects. This raises issues similar to the ones in nucleon propagation: Are large and canceling Lorentz scalar and vector optical potentials predicted for other baryons? Are the sum-rule predictions consistent with experiment and phenomenology? These calculations provide important consistency checks on the sum-rule predictions for nucleons. There are ongoing studies of various baryons; in this review, we will consider hyperon propagation through nuclear matter [22,27], for which there is phenomenology to confront.

In addition to baryons, one can consider meson propagation at finite density. The case of vector mesons, is particularly interesting and important, in part, because changes in vector-meson masses have been proposed as an explanation for anomalies in electron scattering [52,53] and K^+ scattering data [54], and, in part, because measurements of dilepton pairs emerging from a heavy-ion collision may provide a more-or-less direct experimental probe [55,56]. If vector mesons are created in the medium, they have a nonzero amplitude for decay into two leptons. Since the electromagnetic interaction is weak, the dilepton pair has a good probability of leaving the nucleus unscathed, providing essentially unfiltered information about the vector meson in the nucleus. A number of heavy-ion experiments

have been proposed to look for shifts in vector-meson masses via study of dilepton pairs [55]. In addition, the use of the virtual Compton process (γ,dilepton) might provide an even cleaner probe of the vector meson in medium [56].

II. REVIEW OF VACUUM FORMALISM

A. Overview

In this section, we review the QCD sum-rule approach as applied to the calculation of hadronic masses in the vacuum, *i.e.*, at zero density. An overview of QCD sum-rule methods and results is provided by the recent book edited by Shifman [10], which includes a collection of reprints along with an up-to-date commentary by the editor and others. Since thorough technical reviews exist, we will focus here on features and analogies that make the generalization to the finite-density problem more straightforward and plausible.

B. The Nature of QCD Sum Rules

The designation "QCD sum rules" is often confusing to the nonexpert. In quantum chromodynamics there are various relations referred to as "sum rules," which have nothing to do with the approach we are concerned with here. For example, there are many sum rules that originated with the parton model: Bjorken sum rule, Ellis-Jaffe sum rule, Gottfried sum rule, *etc.* These typically are relations that manifest a constraint on properties of constituents, such as a conservation law. So, for example, all of the momenta (or strangeness) of the constituents must add up to the total momentum (or strangeness).

In contrast, the type of sum rule we are concerned with here is typified by the dipole sum rules of atomic and nuclear physics. Consider photo-absorption by nuclei. The cross section for the excitation of a final state $|\nu\rangle$ by a photon of energy E is (see, for example, Ref. [57])

$$\sigma_\nu(E) = \frac{4\pi^2 e^2}{\hbar c}(E_\nu - E_0)|\langle\nu|D|0\rangle|^2\delta(E - E_\nu + E_0) \ . \tag{2.1}$$

Here D is the dipole operator for E1 radiation in the z direction, which can excite states that have isovector, $J^\pi = 1^-$ quantum numbers. The total cross section for dipole absorption is found by summing over all possible final states and integrating over the energy:

$$\sigma_{\rm tot} = \int_0^\infty dE \sum_\nu \sigma_\nu(E) = \frac{4\pi^2 e^2}{\hbar c} S_1(D) \ , \tag{2.2}$$

where we have defined the *energy-weighted dipole sum rule*

$$S_1(D) \equiv \sum_\nu (E_\nu - E_0)|\langle\nu|D|0\rangle|^2 \ . \tag{2.3}$$

On the other hand, one can use completeness to rewrite $S_1(D)$ as a ground-state matrix element involving D and the Hamiltonian H [57]:

$$S_1(D) = \tfrac{1}{2}\langle 0|[D,[H,D]]|0\rangle \ . \tag{2.4}$$

The double commutator can be directly evaluated in a theoretical model. For example, if we model the nucleon-nucleon interaction to have no exchange mixtures and to be velocity independent, we can evaluate the commutator immediately (and obtain the TRK result) [57]. More generally, we will have to do an approximate model calculation.

The sum rule consists of the equality between the ground-state matrix element, which can be approximated in a theoretical model, and the sum over excited states, which is a weighted average of experimental observables. The operator D is the analog of the interpolating field in QCD sum rules, while $\sum_\nu \sigma_\nu(E)$ is the analog of the spectral density. Various general observations can be made about these sum rules, which will carry over to analogous QCD sum rules:

1. If one uses different multipole operators, different intermediate states (corresponding to different quantum numbers) are excited. Thus one can investigate isoscalar or isovector states, and monopole, dipole, quadrupole excitations, and so on. For QCD sum rules, we choose an interpolating field that excites a particular channel of intermediate states which includes the hadron of interest (*e.g.*, nucleon or ρ meson).

2. One can choose an operator that doesn't actually correspond to a physically realizable experiment. The matrix elements in the sum over states may not be measured in a real experiment, but a sum rule can relate their values to a theoretical calculation. Furthermore, in QCD there is the additional possibility of a *numerical* experiment to establish this sum, through lattice calculations.

3. One can choose different weighting functions for the sum rule and thereby change the weight in the sum over states (*cf.* dispersion relation). For example, given a single-particle operator F, we can derive the sum rule [57]

$$S_k(D) \equiv \sum_\nu (E_\nu - E_0)^k |\langle \nu|F|0\rangle|^2 = \langle 0|F(H-E_0)^k F|0\rangle \ . \tag{2.5}$$

   Thus one has some freedom to adjust how important particular states are in the sum. In QCD sum rules, one finds that the weighting resulting from a *Borel transform* of the correlation function is particularly effective.

4. Certain states may dominate the sum. The collective excitations known as giant resonances are a case in point. If a giant resonance largely dominates ("saturates") the sum over states of the strength, then one obtains from the sum rule a direct connection between some experimental properties of this resonance and a theoretical calculation. Furthermore, we can use the freedom of changing the weighting function to improve the dominance of the state of interest. Note that one does not learn everything about the states, just certain matrix elements.

In QCD sum rules, we include an external parameter (the Borel mass) that adjusts the weighting function. We seek a range of parameter values for which we find dominance by the hadronic state of interest as well as a reasonable theoretical approximation to the ground-state expectation value. That the latter goal is attainable in QCD is not obvious. In the next section, we outline how it could be possible.

C. Basic Strategy

Shifman has compared the strategy of the sum-rule approach to the quantum-mechanical many-body problem of some external objects injected into a complicated medium [58]. (This is what a Green's function does, for example.) The picture is that, over time, the external objects will interact with themselves *and* with the medium and will eventually develop into a stationary state or, more generally, an approximate stationary state with a finite lifetime. In many-body systems, the latter is known as a quasiparticle, while the analog in particle physics is a hadronic resonance (such as the ρ meson). The objects can also develop into many other complicated states with the same quantum numbers, but our goal is to isolate some properties of the quasiparticle, such as its energy or amplitude for creation (*i.e.*, the spectral weight).[1]

The medium is too complicated to solve microscopically. At best we might know some coarse, averaged thermodynamic properties. Ideally we would simply evolve the system for large times (but still less than the quasiparticle lifetime), so that the quasiparticle develops and becomes dominant. To do this, however, we would need to calculate the interactions of the objects with the medium and the reaction of the medium back on the objects. By assumption we cannot calculate these details.

Suppose, however, that the characteristic time for the medium to react is long compared to the characteristic time for the external objects. Then there is a window in time corresponding to some cycles of the developing bound state during which the medium is essentially frozen, and the objects see only coarse properties. Yet it is enough time that some basic features of the bound state are established. Then we have a chance to extrapolate from relatively short times!

The key feature is the distinction in time scales. If certain aspects of the physics are established at short times, they can persist to the measurement at long times, even though the system appears very different. This is reminiscent of the parton model, as applied to deep-inelastic scattering. In the infinite-momentum frame, one sees a clear separation of time scales: the interaction of the photon with the constituent partons and the interaction of the constituents among themselves. One can calculate the cross section for inclusive scattering using partons and get the right answer, even though there are strong (confining) final-state interactions and the partons end up in detectors as hadrons.

The analogy to QCD sum rules is that the medium is the QCD vacuum and the external objects are valence quarks (or gluons) with definite quantum numbers created by external currents (which we specify). We are able to calculate the behavior only at relatively short times[2] using the operator product expansion. The coarse properties of the medium seen by the valence quarks are characterized by vacuum matrix elements of quark and gluon fields, called condensates. The question of whether there really is a favorable relation of the time scale characteristic of the valence quarks and that of the vacuum fluctuations is not clearcut

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[1]Note that the latter by itself is not generally an observable.

[2]Actually, we will look effectively at short *imaginary* times.

in QCD, and seems to depend strongly on the channel [59].

However, for light-quark hadrons, the existence of two mass (or time) scales has been postulated in various contexts (for example, Georgi and Manohar's chiral quark model [60]). The vacuum response is characterized by times of order $1/\Lambda_{\rm QCD}$. The shorter time scale for the light-quark hadrons might be set by the chiral symmetry breaking scale. So one is comparing roughly $\Lambda_{\rm QCD}$ to the mass of the ρ meson or the nucleon. While this is not a huge difference, it may be sufficient to establish some dominant features of the ground state from the physics of chiral symmetry breaking before confining interactions play a direct role in the bound state. (Compare to a quantum-mechanical system in a large box. The box serves to discretize the spectrum, but does not change the total cross section if we average over nearby levels [10].) This scenario is consistent with a picture of constituent quarks forming first, then becoming weakly bound at later times. Indeed, one of the basic physical predictions of QCD sum rules is that chiral symmetry breaking, as manifested by the chiral condensate, predominantly sets the scale for the masses of light-quark hadrons.

The generalization of our analogy to finite density simply means that we inject valence quarks into a modified medium, the nuclear matter ground state. The background fields (condensates) seen by the injected quarks will be somewhat different, which would then be reflected in the predictions for self-energies at finite density. If we focus on quantities that are not strongly dependent on physics at large time scales, the successes of the vacuum sum rules should carry over to finite density.

D. Field Conventions

In this section we review the basic QCD field operators to establish our notation and conventions. We start with the fields for the up, down, and strange quarks, $u_{a\alpha}$, $d_{a\alpha}$, and $s_{a\alpha}$, where we use $a, b, c, \ldots = 1$–3 for quark color indices and $\alpha, \beta, \gamma, \ldots = 1$–$4$ for Dirac indices. Due to isospin symmetry, we often do not distinguish between up and down quark flavors; we use $q_{a\alpha}$ to refer to either an up or down quark field.

The gluon field is denoted A^A_λ, where we use $A, B, C, \ldots = 1$–8 for gluon color indices and $\lambda, \mu, \nu, \ldots = 0$–$3$ for Lorentz indices. A matrix form of the gluon field is obtained by multiplying the color components of the gluon field by the generators of the SU(3) Lie algebra:

$$\mathcal{A}^\mu_{ab} \equiv A^{A\mu} t^A_{ab} \ , \tag{2.6}$$

where $t^A \equiv \lambda^A/2$ are the SU(3) generators in the fundamental representation (λ^A are the Gell-Mann matrices [61]). They satisfy the following relations:

$$[t^A, t^B] = i f^{ABC} t^C \ , \qquad \mathrm{tr}(t^A) = 0 \ , \qquad \mathrm{tr}(t^A t^B) = \tfrac{1}{2}\delta^{AB} \ , \tag{2.7}$$

where f^{ABC} are totally antisymmetric structure constants [61] and tr denotes a trace over quark color indices.

It is useful to introduce the gluon field tensor, which can be defined as

$$\mathcal{G}_{\mu\nu} \equiv G^A_{\mu\nu} t^A \equiv D_\mu \mathcal{A}_\nu - D_\nu \mathcal{A}_\mu \ , \tag{2.8}$$

where $D_\mu \equiv \partial_\mu - ig_s\mathcal{A}_\mu$ is the covariant derivative (g_s is the quark-gluon coupling constant). Useful identities that follow from Eqs. (2.7) and (2.8) are

$$\mathcal{G}_{\mu\nu} = \frac{i}{g_s}[D_\mu, D_\nu] \ , \tag{2.9}$$

$$G^A_{\mu\nu} = \partial_\mu A^A_\nu - \partial_\nu A^A_\mu + g_s f^{ABC} A^B_\mu A^C_\nu \ . \tag{2.10}$$

We also define the dual of the gluon field tensor,

$$\widetilde{G}^{A\kappa\lambda} \equiv \tfrac{1}{2}\epsilon^{\kappa\lambda\mu\nu} G^A_{\mu\nu} \ . \tag{2.11}$$

The quark fields are coupled to the gluon field by replacing ordinary derivatives with covariant derivatives in the free quark Lagrangians. Thus the quark fields q and $\overline{q}$ (with current quark mass m_q) satisfy the following equations of motion:

$$(i\not{D} - m_q)q = 0 \ , \qquad\qquad \overline{q}(i\overleftarrow{\not{D}} + m_q) = 0 \ , \tag{2.12}$$

where we define $\overleftarrow{D}_\mu \equiv \overleftarrow{\partial}_\mu + ig_s\mathcal{A}_\mu$. These equations of motion will be used in the subsequent discussion to simplify operator matrix elements. In nucleon sum rules, the contributions from finite up and down current quark masses are numerically small; therefore, we neglect these contributions.

E. Ingredients of a QCD Sum-Rule Calculation

In this section we review the ingredients of a QCD sum-rule calculation. Since our primary focus in this review is on the nucleon sum rule in medium, we will use the nucleon sum rule in vacuum as our example here. Particular details of other sum rules can be found in Refs. [8–10].

Conventional treatments of QCD sum rules start with a momentum-space correlation function (also called a correlator) of color-singlet currents. For example, the analysis of the nucleon mass [6] starts with the time-ordered correlation function $\Pi(q)$ defined by

$$\Pi_{\alpha\beta}(q) \equiv i \int d^4x \, e^{iq\cdot x} \langle 0|T\eta_\alpha(x)\overline{\eta}_\beta(0)|0\rangle \ , \tag{2.13}$$

where $|0\rangle$ is the physical nonperturbative vacuum state, and η_α is an interpolating field with the spin and isospin of a nucleon, but with indefinite parity. We have exhibited the Dirac indices α and β. Here we choose to use the interpolating field for the proton proposed by Ioffe [6,62]:

$$\eta_p = \epsilon_{abc}(u^T_a C\gamma_\mu u_b)\gamma_5\gamma^\mu d_c \ , \tag{2.14}$$

where u_a and d_a are up and down quark fields (a is a color index), T denotes a transpose in Dirac space, and C is the charge-conjugation matrix.

The choice of interpolating field for the nucleon is motivated by the goals of maximizing the coupling to the nucleon intermediate state relative to other (continuum) states while

minimizing the contributions of higher-order corrections from the operator product expansion. Furthermore, we want the two invariant functions [see Eq. (2.20)] to be more-or-less equally dominated by the nucleon contribution. These goals plus the simplicity of an interpolating field with no derivatives and the minimum number of quark fields motivate η_p [6,62]. In Sec. III F the sensitivity to this choice for finite-density sum rules is tested.

The fundamental physical object in the sum-rule analysis is the *spectral density* of the interpolating fields. For our example here, we can identify spectral densities corresponding to the two time orderings:

$$\rho_{\alpha\beta}(q) = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle 0|\eta_\alpha(x)\overline{\eta}_\beta(0)|0\rangle \quad , \tag{2.15}$$

$$\widetilde{\rho}_{\alpha\beta}(q) = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle 0|\overline{\eta}_\beta(0)\eta_\alpha(x)|0\rangle \quad . \tag{2.16}$$

The spectral densities of the correlator follow from the assumed dispersion relation (ignoring, for simplicity, any subtractions, which are not relevant here):

$$\Pi_{\alpha\beta}(q) = -\int_{-\infty}^{+\infty} dq_0' \left[\frac{\rho_{\alpha\beta}(q')}{q_0 - q_0' + i\epsilon} + \frac{\widetilde{\rho}_{\alpha\beta}(q')}{q_0 - q_0' - i\epsilon}\right] , \tag{2.17}$$

where $q'_\mu = (q'_0, \mathbf{q})$. From this expression, one sees that the spectral densities are equal, up to overall constants, to the discontinuity of the correlator across the real axis.

After inserting intermediate states between the currents in Eqs. (2.15) and (2.16), we can extract the x dependence of the η field and do the x integral, yielding

$$\rho_{\alpha\beta}(q) = (2\pi)^3 \sum_n \delta^{(4)}(q - P_n)\langle 0|\eta_\alpha(0)|n\rangle\langle n|\overline{\eta}_\beta(0)|0\rangle \ , \tag{2.18}$$

and a similar expression for $\widetilde{\rho}_{\alpha\beta}(q)$ [but with $\delta^{(4)}(q + P_n)$]. Here P_n^μ is the four-momentum of the state $|n\rangle$. For fixed three-momentum $\mathbf{q}$, the spectral function measures the intensity at which energy is absorbed from the current at different frequencies. Thus it is analogous to the absorption cross section $\sum_\nu \sigma_\nu(E)$ in Eq. (2.1). (In some finite-density sum rules, we can also have scattering from the medium.)

Lorentz covariance and parity invariance imply that the Dirac structure of $\rho_{\alpha\beta}(q)$ is of the form [63,61]

$$\rho_{\alpha\beta}(q) = \rho_s(q^2)\delta_{\alpha\beta} + \rho_q(q^2)\not{q}_{\alpha\beta} \ , \tag{2.19}$$

and similarly for $\widetilde{\rho}$, so that $\Pi(q)$ has an analogous form:

$$\Pi_{\alpha\beta}(q) = \Pi_s(q^2)\delta_{\alpha\beta} + \Pi_q(q^2)\not{q}_{\alpha\beta} \ , \tag{2.20}$$

where Π_s and Π_q are Lorentz scalar functions of q^2 only. An independent sum rule for each scalar function, $\Pi_s(q^2)$ and $\Pi_q(q^2)$, will be constructed.

Charge-conjugation invariance at zero density implies that the invariant spectral functions $\rho_i(q^2)$ and $\widetilde{\rho}_i(q^2)$ (where $i = \{s, q\}$) are, in fact, identical, up to an overall sign. This will *not* be true at finite density. Thus, for the vacuum case, one need integrate only over positive energy. This means that the integral over energy in Eq. (2.17) can be transformed

to one over $q'^2 \equiv s$. The end result is the familiar dispersion relation for each invariant function:

$$\Pi_i(q^2) = \int_0^\infty ds\, \frac{\rho_i(s)}{s - q^2} + \text{polynomial} \ . \tag{2.21}$$

At finite density we will keep the dispersion relations as integrals over energy.

The dispersion relation in Eqs. (2.17) or (2.21) embodies a particular weighting of the spectral density, integrated over all energies. When the correlator is evaluated for spacelike external momenta ($q^2 < 0$), the weighting function is analytic and serves to smear smoothly the spectral density in energy. It is also clear that one could devise a different weighting function that goes to zero sufficiently fast at large energy that the weighted integral is assured to be finite (and no subtraction is needed). This weighted sum over states is one half of our sum rule.

The other half of the sum rule is a direct calculation of the correlator using the operator product expansion (OPE), which provides a QCD approximation to the correlator that is applicable at large spacelike q^2. The OPE is a useful tool for extracting phenomenological information from renormalizable quantum field theories. The central idea is that the time-ordered product of two local (elementary or composite) operators at short distances can be expanded in terms of a complete set of regular local operators $\widehat{O}_n(0)$ [29,30]:

$$TA(x)B(0) \overset{x\to 0}{=} \sum_n C_n^{AB}(x)\widehat{O}_n(0) \ . \tag{2.22}$$

The c-number coefficients C_n^{AB} of the expansion are called Wilson coefficients. In this expansion, the singularities at short distances are factored out from the regular operators, and the terms in the expansion are organized in decreasing order of singularity. Formally, the OPE has only been proven in perturbation theory; the validity of the OPE in the presence of nonperturbative effects is a complex issue.

There has been a series of papers discussing the nature of the OPE beyond perturbation theory [64–66]. It was shown that, to apply unambiguously the OPE, one must define the coefficient functions and composite operators by introducing an auxiliary parameter, the normalization point μ:

$$TA(x)B(0) \overset{x\to 0}{=} \sum_n C_n^{AB}(x,\mu)\widehat{O}_n(0,\mu) \ . \tag{2.23}$$

Physics from momenta above μ are put in the coefficients C_n^{AB}, while physics from momenta below μ are put into the operators $\widehat{O}_n$. Both the coefficient functions and the operators contain, in general, nonperturbative as well as perturbative contributions. However, in practical applications of the OPE (in particular, in the QCD sum-rule formalism), one usually applies a simplified version. Namely, the Wilson coefficients are evaluated in perturbation theory, while the nonperturbative effects are contained entirely in the vacuum expectation values of composite operators, which are assumed to contain no perturbative contributions.

This simplified version of the OPE is justified in part by the phenomenological success of QCD sum rules. In Ref. [65], the reason behind this success is attributed to the following: There seems to be a range in μ in which μ is large enough with respect to Λ_{QCD} (the QCD

scale parameter) so that nonperturbative corrections to the coefficients are small and can be neglected, but small enough so that the values of the condensates are quite insensitive to variations in μ. In other words, there seems to be a "window" in QCD where the simplified version of the OPE applies. Perturbative corrections to the operators can be taken into account in the leading-logarithmic approximation through anomalous-dimension factors [5] (these will be implicit in the rest of our discussion).

In momentum space, the OPE correlator takes the general form

$$\Pi(Q^2) = \sum_n C_n^i(Q^2)\langle\widehat{O}_n\rangle_{\mathrm{vac}} \ , \tag{2.24}$$

where the $C_n^i(Q^2)$ are c-number functions, calculated in QCD perturbation theory, and the $\langle\widehat{O}_n\rangle_{\mathrm{vac}} \equiv \langle 0|\widehat{O}_n|0\rangle$ are vacuum expectation values of QCD operators—the condensates. The expansion is in inverse powers of Q^2 and thus is useful in the deep spacelike region, for which $Q^2 \equiv -q^2$ is large and positive.

Equating the OPE correlator and the spectral expansion, each evaluated at spacelike q^2, gives us a sum rule. The equivalence exploits the underlying principle of *duality*. This term is used in various contexts; for our purposes, it is most clearly expressed in terms of smeared spectral densities. On the one hand, the spectral density is a sum of matrix elements with physical intermediate states, including the hadron of particular interest. On the other hand, one could calculate the spectral density in perturbation theory with a basis of quarks and gluons.[3] Superficially these have little in common; nevertheless, if these spectral densities are sufficiently smeared, one should obtain the same result with either representation. (See the harmonic oscillator example in Sec. II F.)

Smearing broadly with an analytic function can be thought of as focusing on the short Euclidean time parts of the spectral density, which makes it amenable to approximation by an OPE. The Euclidean correlator on the lattice in imaginary time is a clear example [see Eq. (6.2)]. We note that the choice of smearing function is rather arbitrary and, furthermore, that some choices are more useful than others. The experience with QCD sum rules is that a near optimal choice corresponds to a Gaussian weighting function (in energy). We can achieve this weighting starting from the correlator by applying a Borel transform.

Given a function $f(Q^2)$, the Borel transform $\mathcal{B}[f(Q^2)] \equiv \widehat{f}(M^2)$ can be defined in practice by [5]

$$\widehat{f}(M^2) \equiv \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=M^2}} \frac{(Q^2)^{n+1}}{n!}\left(-\frac{d}{dQ^2}\right)^n f(Q^2) \ , \tag{2.25}$$

which depends on the "Borel mass" M. Table I lists the Borel transforms of the most commonly encountered functions in sum-rule applications.[4] Expressions involving the running coupling α_s are derived using the one-loop expression:

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[3]The OPE effectively gives corrections to the perturbative spectral density due to nonperturbative effects.

[4]Some additional useful results can be derived by using integral representations of functions that

$$\alpha_s(Q^2) \simeq \frac{4\pi}{b\ln(Q^2/\Lambda^2_{\mathrm{QCD}})} \ , \tag{2.26}$$

and are valid to lowest order in α_s.

One notes from Table I that any simple polynomial in Q^2 is eliminated by the Borel transform. This has two useful (and related) consequences: The subtraction terms accompanying the dispersion relation and any divergent (or renormalized) polynomials from the OPE are simultaneously eliminated. It is also evident that the higher-order terms in the OPE, which contain inverse powers of Q^2, are factorially suppressed by the Borel transform.

After applying the Borel transform to the dispersion relation of each invariant function, we obtain the desired weighting:

$$\widetilde{\Pi}_i(M^2) = \int_0^\infty ds\, e^{-s/M^2} \rho_i(s) \ . \tag{2.27}$$

For M near the mass of the nucleon, higher-mass contributions to the integral are exponentially suppressed.

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isolate the Q^2 dependence in the integrand in the form $f(Q^2) = e^{-zQ^2}$ with $z > 0$. The Borel transform is found using $\widehat{f}(M^2) = M^2\delta(zM^2 - 1)$.

TABLES

TABLE I. Borel transforms of common functions, where $k = 1, 2, 3, \ldots$, $m = 0, 1, 2, \ldots$, and ϵ is not necessarily integral. Transforms involving α_s are given to leading order.

$f(Q^2)$	$\widehat{f}(M^2)$
$\dfrac{1}{(Q^2)^k}$	$\dfrac{1}{(k-1)!(M^2)^{k-1}}$
$\dfrac{\alpha_s(Q^2)}{(Q^2)^k}$	$\dfrac{\alpha_s(M^2)}{(k-1)!(M^2)^{k-1}} + \cdots$
$(Q^2)^m$	0
$(Q^2)^m \ln Q^2$	$(-1)^{m+1} m!(M^2)^{m+1}$
$\alpha_s(Q^2)(Q^2)^m \ln Q^2$	$\alpha_s(M^2)(-1)^{m+1} m!(M^2)^{m+1} + \cdots$
$\dfrac{1}{(Q^2+\mu^2)^\epsilon}$	$\dfrac{1}{\Gamma(\epsilon)(M^2)^{\epsilon-1}} e^{-\mu^2/M^2}$

The correlator in spectral form can be evaluated by introducing a phenomenological model for the spectral density. The lowest-energy contribution to the spectral function is from the nucleon pole. Its contribution can be constructed from the matrix element

$$\langle 0|\eta(0)|q\rangle = \lambda_N u(q) \ , \tag{2.28}$$

where $|q\rangle$ is a one-nucleon state with four-momentum q^μ (with $q^2 = M_N^2$) and $u(q)$ is a Dirac spinor for the nucleon. Contributions to the spectral functions from higher-mass states are roughly approximated using the leading terms in the OPE, starting at a threshold s_0 [see Eqs. (2.53) and (2.54)].

Equating the Borel transforms of the OPE and phenomenological descriptions yields relations—sum rules—for each invariant function. To summarize, the general recipe for a QCD sum-rule calculation is:

1. identify an appropriate interpolating field for the hadron of interest and construct a correlator;
2. identify the tensor structure and invariant functions of the correlator;
3. write dispersion relations for each invariant function with a spectral ansatz;
4. construct the OPE for each invariant function;
5. convert to Borel weighting;
6. match and extract parameters of the ansatz.

Examples of this procedure are given in the next two sections.

F. Harmonic Oscillator Analogy

To illustrate the QCD sum-rule approach for nonexperts, it is useful to look at a simple, solvable problem from ordinary quantum mechanics. Indeed, it is difficult to believe that the approach is feasible without some direct evidence. Illustrative calculations for various three-dimensional quantum mechanical potentials are given in the literature (see, for example, Refs. [59] and [67]). There are also examples in model field theories, which provide additional verifications of the methods.

Here we will go through the three-dimensional harmonic oscillator. The harmonic oscillator is a particularly nice example, because, besides the advantage of being exactly solvable, it has features analogous to QCD: confinement and asymptotic freedom. The confinement analogy is obvious: the potential goes to infinity so that there are only bound states, no continuum states. (One should imagine the one-body harmonic oscillator problem as arising from a two-particle problem, with a harmonic oscillator potential between them. This, of course, can be reduced to a one-body problem in an external potential.) The analog of asymptotic freedom is less obvious, but follows because the potential is nonsingular at the origin, which means that the kinetic energy will dominate the potential energy at short times.

The Hamiltonian for the harmonic oscillator is

$$H = \frac{1}{2m}\mathbf{p}^2 + \frac{m\omega^2}{2}\mathbf{x}^2 \ , \tag{2.29}$$

where m is the mass and ω is the oscillator parameter. Complete information about the system is given by the energies E_n and wave functions $\psi_n(\mathbf{x})$ of the bound states, but this is far more than we can calculate with sum-rule methods. Instead we focus on the ground-state energy and wave function at the origin. The latter is the analog to something calculated in QCD sum rules: an amplitude for finding quarks in the bound state on top of each other. Thus we will deal with s states only in the sequel.

Our goal with the sum rules is to calculate the energy $[E_0 = (3/2)\omega]$ and wave function at the origin $[|\psi_0(0)|^2 = (m\omega/\pi)^{3/2}]$ of the lowest bound state. We start with the coordinate-space Green's operator or resolvent [67]:

$$G(\mathbf{x},\mathbf{y};z) \equiv \langle \mathbf{x}|\frac{1}{z-H}|\mathbf{y}\rangle = \sum_n \frac{\psi_n(\mathbf{x})\psi_n^*(\mathbf{y})}{z-E_n} \ , \tag{2.30}$$

where the second equality is obtained by inserting a complete set of eigenstates of H. In the complex z plane, there are poles at $z = E_n$. We will evaluate G for $z = -E < 0$, away from the poles ("spacelike"). The analog of the QCD correlator follows by setting $\mathbf{x} = \mathbf{y} = 0$:

$$M_1(E) \equiv -G(\mathbf{x},\mathbf{y};-E)\Big|_{\mathbf{x}=\mathbf{y}=0} = \sum_n \frac{|\psi_n(0)|^2}{E+E_n} \ . \tag{2.31}$$

But this doesn't converge (*cf.* the need for subtractions), so we look at the *derivative* instead:

$$M_2(E) \equiv \frac{d}{dE}G(\mathbf{x},\mathbf{y};-E)\Big|_{\mathbf{x}=\mathbf{y}=0} = \sum_n \frac{|\psi_n(0)|^2}{(E+E_n)^2} \ . \tag{2.32}$$

For the harmonic oscillator, E_n and $|\psi_n(0)|^2$ are given by

$$E_n = (\tfrac{3}{2} + 2n)\omega \qquad \text{and} \qquad |\psi_n(0)|^2 = \frac{(2n+1)!!}{2^n n!}\left(\frac{m\omega}{\pi}\right)^{3/2} . \tag{2.33}$$

Equation (2.32) is half of our candidate sum rule: a sum over states.

We can also evaluate $M_2(E)$ directly in perturbation theory after constructing the Born series for G. The first-order result, for a general central potential $V(r)$, is

$$M_2(E) = \left(\frac{m^3}{8\pi^2 E}\right)^{1/2} [1 - 4m \int_0^\infty dr\, r\, e^{-\sqrt{8mE}\, r}\, V(r) + \cdots] , \tag{2.34}$$

and for the harmonic oscillator is

$$M_2(E) = \left(\frac{m^3}{8\pi^2 E}\right)^{1/2} \left(1 - \frac{3}{16}\frac{\omega^2}{E^2} + \cdots \right) . \tag{2.35}$$

Note that, for large E, the free term dominates and the contribution from the potential is a controlled correction. Equating the two representation of $M_2(E)$ gives us our sum rule.

Can we find values of E for which Eq. (2.35) is accurately calculated with one or two corrections *and* Eq. (2.32) is dominated by the $n = 0$ term? For the harmonic oscillator, taking $E \simeq \omega$ in Eq. (2.35) gives the exact answer to within a few percent. However, the contribution of the ground state in Eq. (2.32) is only about 1/3 of the total and even summing the first eight states only gives about 3/4! It would appear that our sum rule is useless.

The solution is to use a better weighting function in the sum, such as an exponential, to create a new, improved sum rule. We can reach such a sum rule by applying a Borel transform with respect to E to each representation of $M_2(E)$.[5] The resulting sum rule is [67]:

$$\begin{aligned}\sum_{n=0}^{\infty} |\psi_n(0)|^2 e^{-E_n/\epsilon} &= \left(\frac{m\epsilon}{2\pi}\right)^{3/2} [1 - 4m \int_0^\infty dr\, r\, e^{-2m\epsilon r^2} V(r) + \cdots] \\ &= \left(\frac{m\epsilon}{2\pi}\right)^{3/2} \left(1 - \frac{\omega^2}{4\epsilon^2} + \cdots\right) .\end{aligned} \tag{2.36}$$

This function can be recognized as a special case of the time-dependent Green's function, continued to imaginary time $-i/\epsilon$. Note that for large ϵ a large number of levels make an important contribution to the sum, whereas as we go to small ϵ the lowest level dominates increasingly.

Now we try the harmonic oscillator again, with $\epsilon = \omega$, and again we find that first-order perturbation theory gives the exact answer to within five percent. But now the ground state contribution is 80% of the exact sum, and the first two levels together give 97%! So it is plausible that this sum rule will let us extract information about the ground state from low-order perturbation theory.

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[5]Use Eq. (2.25) with $Q^2 \to E$ and $M^2 \to \epsilon$.

It is instructive to introduce the spectral density

$$\rho^{\rm osc}(E) = \pi \sum_{n=0}^{\infty} |\psi_n(0)|^2 \delta(E - E_n) \ , \tag{2.37}$$

where the sum is over a complete set of energy eigenstates. We can write the left-hand side of Eq. (2.36) as

$$\sum_{n=0}^{\infty} |\psi_n(0)|^2 e^{-E_n/\epsilon} = \frac{1}{\pi} \int_{-\infty}^{+\infty} dE \, e^{-E/\epsilon} \, \rho^{\rm osc}(E) \ . \tag{2.38}$$

The free spectral density follows from substituting the free Hamiltonian and plane-wave states normalized in volume Ω:

$$\rho^{\rm free}(E) = \pi \sum_{\mathbf{k}} \frac{1}{\Omega} \delta(E - \mathbf{k}^2/2m) \longrightarrow \left(\frac{m^3}{2\pi^2}\right)^{1/2} E^{1/2} \theta(E) \ . \tag{2.39}$$

It would not appear from comparing Eqs. (2.37) and (2.39) that $\rho^{\rm free}$ and $\rho^{\rm osc}$ have much in common.

However, if we smear the spectral densities in energy, we find some similarities. If we rewrite Eq. (2.36) in terms of integrals over spectral densities and transfer the free contribution to the left-hand side, it takes the form

$$\frac{1}{\pi} \int_{-\infty}^{\infty} dE \, [\rho^{\rm osc}(E) - \rho^{\rm free}(E)] e^{-E/\epsilon} = \frac{1}{\sqrt{\epsilon}} \sum_{n=0}^{\infty} \frac{A_n}{\epsilon^{2n}} \ . \tag{2.40}$$

The terms on the right-hand side, which are suppressed for large ϵ by powers of $1/\epsilon$, are usually referred to as "power corrections." If we take the limit $\epsilon \to \infty$, we find that the integrated spectral densities are equal:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} dE \, [\rho^{\rm osc}(E) - \rho^{\rm free}(E)] = 0 \ . \tag{2.41}$$

This is referred to as a "global duality" relation.

This similarity between the spectral densities can be extended to more local relations. That is, if we integrate from zero to the midpoint between the first and second levels, we get quite close to the same result:

$$\frac{1}{\pi} \int_{0}^{5\omega/2} dE \, \rho^{\rm free}(E) \simeq \frac{1}{\pi} \int_{0}^{5\omega/2} dE \, \rho^{\rm osc}(E) = \left(\frac{m\omega}{\pi}\right)^{3/2} \ . \tag{2.42}$$

Similar results are obtained for other levels and if the integrands include a factor of E. This is called "local duality" between the bound and free states. Each bound state eats up its part of the free spectral density, leaving the magnitude of the integral unchanged.

The trick of the sum rules is to exploit this duality. In particular, we approximate the true spectral density as the contribution from the lowest state, which is what we seek to determine, and a contribution from higher states approximated by duality. Thus we use $\rho^{\rm free}$ starting at some threshold s_0, which becomes a parameter to be determined as well. Then we have

$$|\psi_0(0)|^2 e^{-E_0/\epsilon} + \frac{1}{\pi}\int_{s_0}^{\infty} dE\, \rho^{\rm free}(E)\, e^{-E/\epsilon} = \left(\frac{m\epsilon}{2\pi}\right)^{3/2}\left(1 - \frac{\omega^2}{4\epsilon^2} + \frac{19}{480}\frac{\omega^4}{\epsilon^4} + \cdots\right)\,, \qquad (2.43)$$

or, putting the continuum contribution on the right hand side,

$$|\psi_0(0)|^2 e^{-E_0/\epsilon} = \frac{1}{\pi}\int_{0}^{s_0} dE\, \rho^{\rm free}(E)\, e^{-E/\epsilon} + \left(\frac{m\epsilon}{2\pi}\right)^{3/2}\left(- \frac{\omega^2}{4\epsilon^2} + \frac{19}{480}\frac{\omega^4}{\epsilon^4} + \cdots\right)\,. \qquad (2.44)$$

The parameters to be extracted from the sum rule are E_0, $|\psi_0(0)|^2$, and s_0. One notes that ϵ is an auxiliary parameter in the sum rule. If the sum rule were perfect, we could pick any value of ϵ. However, each side is approximate, so at best we expect a *region* in ϵ for which each side is well approximated. This fiducial region is determined in practice by insisting that the errors due to the rough model of the higher states and the contributions of omitted higher power corrections are both small.

There are various methods for extracting the parameters. The ratio method can be useful if we have two sum rules that we can divide to remove the dependence on the wave function. In the present case, differentiating with respect to $-1/\epsilon$ generates another sum rule. Dividing the two sum rules yields an estimate for E_0:

$$E_0 = \frac{\frac{1}{\pi}\int_0^{s_0} dE\, \rho^{\rm free}(E)\, E e^{-E/\epsilon} + (\frac{m\epsilon}{2\pi})^{3/2}\epsilon(\frac{3}{2} + \frac{\omega^2}{8\epsilon^2} - \frac{19}{192}\frac{\omega^4}{\epsilon^4} + \cdots)}{\frac{1}{\pi}\int_0^{s_0} dE\, \rho^{\rm free}(E)\, e^{-E/\epsilon} + (\frac{m\epsilon}{2\pi})^{3/2}(-\frac{\omega^2}{4\epsilon^2} + \frac{19}{480}\frac{\omega^4}{\epsilon^4} + \cdots)}\,. \qquad (2.45)$$

We can plot this for various values of s_0 and choose the flattest curve in the fiducial interval (say $0.6 < \omega/\epsilon < 1.2$) to determine E_0. With just two or three terms, results close to $E_0 = (3/2)\omega$ are found, with $s_0 \simeq (5/2)\omega$. We can then return to the original sum rule and extract $|\psi_0(0)|^2$.

There are still some subjective elements in this ratio analysis because s_0 is not determined very well. An alternative approach is simply to treat the sum rule as an optimization problem for E_0, $|\psi_0(0)|^2$, and s_0 in a specified interval where the sum rule should work (results should be relatively insensitive to the precise boundaries). One finds that accurate and more stable results are obtained this way, particularly if the continuum estimate is improved by adding the first-order correction to $\rho^{\rm free}$. The QCD analog of the optimization method is described in Sec. III F and will be our method of choice for quantitative analysis of the QCD sum rules.

The moral of the harmonic oscillator analogy is that it *is* possible to construct a sum rule that can be accurately calculated in low-order perturbation theory, but which is also largely saturated by the lowest physical state. The key underlying principle is that of duality between the two representations. In QCD sum rules, we exploit the duality between the hadronic description (the physical spectral density) and the quark-gluon description from the OPE. Note that the main source of error for the harmonic oscillator example is the rough model of the spectral density for the higher states. In the QCD case, the situation is better: Only the first resonance is narrow, with higher-state contributions broadened. Therefore a simple continuum model may be quite reasonable.

G. Sum-Rule Results in Vacuum

Now we return to the nucleon correlator in vacuum and put together a QCD sum rule, following the recipe of Sec. II E and the harmonic oscillator example. We start with the OPE for the functions Π_s and Π_q of Eq. (2.20). (A method to calculate Wilson coefficients is described in Sec. III D 5.) With the interpolating field in Eq. (2.14) and keeping operators up through mass dimension six,[6] one obtains [6]

$$\Pi_s^{\mathrm{OPE}}(q^2) = \frac{q^2}{4\pi^2}\ln(-q^2)\langle\overline{q}q\rangle_{\mathrm{vac}} \ , \tag{2.46}$$

$$\Pi_q^{\mathrm{OPE}}(q^2) = -\frac{(q^2)^2}{64\pi^4}\ln(-q^2) - \frac{1}{32\pi^2}\ln(-q^2)\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}} - \frac{2}{3q^2}\langle\overline{q}q\rangle^2_{\mathrm{vac}} \ , \tag{2.47}$$

where all polynomials in q^2, which vanish under the Borel transform, have been omitted. In Eq. (2.47), the four-quark condensates have already been estimated in terms of the square of $\langle\overline{q}q\rangle_{\mathrm{vac}}$; this factorization assumption will be discussed in detail later. The Borel transform can be calculated using the formulas in Table I to obtain

$$\widehat{\Pi}_s^{\mathrm{OPE}}(M^2) = -\frac{1}{4\pi^2}M^4\langle\overline{q}q\rangle_{\mathrm{vac}} \ , \tag{2.48}$$

$$\widehat{\Pi}_q^{\mathrm{OPE}}(M^2) = \frac{1}{32\pi^4}M^6 + \frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}} + \frac{2}{3}\langle\overline{q}q\rangle^2_{\mathrm{vac}} \ . \tag{2.49}$$

Now consider the phenomenological side of the sum rule. Given the form of the correlator in Eq. (2.13), one can simply write down the phenomenological form of a nucleon pole plus higher-energy continuum states:

$$\Pi^{\mathrm{phen}}(q) = -\lambda_N^2\frac{\not{q} + M_N}{q^2 - M_N^2 + i\epsilon} + \text{continuum} \ , \tag{2.50}$$

where M_N is the nucleon mass and λ_N was introduced in Eq. (2.28). In terms of the scalar functions of Eq. (2.20), one has

$$\Pi_s^{\mathrm{phen}}(q^2) = -\lambda_N^2\frac{M_N}{q^2 - M_N^2 + i\epsilon} + \text{continuum} \ , \tag{2.51}$$

$$\Pi_q^{\mathrm{phen}}(q^2) = -\lambda_N^2\frac{1}{q^2 - M_N^2 + i\epsilon} + \text{continuum} \ . \tag{2.52}$$

The continuum contribution is approximated by the equivalent OPE spectral densities, starting at a sharp threshold s_0, yielding

$$\rho_s^{\mathrm{phen}}(s) = \lambda_N^2 M_N\delta(s - M_N^2) - \frac{1}{4\pi^2}s\langle\overline{q}q\rangle_{\mathrm{vac}}\theta(s - s_0) \ , \tag{2.53}$$

$$\rho_q^{\mathrm{phen}}(s) = \lambda_N^2\delta(s - M_N^2) + \left[\frac{1}{64\pi^4}s^2 + \frac{1}{32\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}}\right]\theta(s - s_0) \ . \tag{2.54}$$

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[6] Recall that terms with explicit dependence on m_q are omitted here.

The same continuum threshold s_0 is assumed for simplicity to be relevant for each spectral density. Substituting into Eq. (2.27), one obtains the Borel transform of the phenomenological correlation function:

$$\widehat{\Pi}_s^{\mathrm{phen}}(M^2) = \lambda_N^2 M_N e^{-M_N^2/M^2} - \frac{M^4}{4\pi^2}\langle\overline{q}q\rangle_{\mathrm{vac}}\left(1+\frac{s_0}{M^2}\right)e^{-s_0/M^2} \ , \tag{2.55}$$

$$\widehat{\Pi}_q^{\mathrm{phen}}(M^2) = \lambda_N^2 e^{-M_N^2/M^2} + \frac{M^6}{32\pi^4}\left(1+\frac{s_0}{M^2}+\frac{s_0^2}{2M^4}\right)e^{-s_0/M^2}$$
$$+\frac{M^2}{32\pi^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\mathrm{vac}} e^{-s_0/M^2} \ . \tag{2.56}$$

At this point, one can make the identifications $\widehat{\Pi}_s^{\mathrm{phen}}(M^2) = \widehat{\Pi}_s^{\mathrm{OPE}}(M^2)$ and $\widehat{\Pi}_q^{\mathrm{phen}}(M^2) = \widehat{\Pi}_q^{\mathrm{OPE}}(M^2)$ and solve for M_N by dividing the equations, with the following result:

$$M_N = \frac{2aM^4\left[1-\left(1+\frac{s_0}{M^2}\right)e^{-s_0/M^2}\right]}{M^6\left[1-\left(1+\frac{s_0}{M^2}+\frac{s_0^2}{2M^4}\right)e^{-s_0/M^2}\right]+bM^2\left(1-e^{-s_0/M^2}\right)+\frac{4}{3}a^2} \ , \tag{2.57}$$

where $a = -(2\pi)^2\langle\overline{q}q\rangle_{\mathrm{vac}}$ and $b = \pi^2\langle(\alpha_s/\pi)G^2\rangle_{\mathrm{vac}}$. Perturbative corrections $\sim \alpha_s^n$ can be taken into account through anomalous-dimension factors (see Ref. [68] for formulas).

The principal physical content of the full sum rule, that the scale of the nucleon mass is largely determined by the quark condensate, is manifest in a highly simplified version of the nucleon sum rule. Detailed sum-rule analyses of the nucleon mass have been made by Ioffe [6] and many others [69,8]. Ioffe concluded that the contributions of higher-dimensional condensates and the continuum tend to cancel for values of the Borel mass in the vicinity of M_N, such that meaningful predictions can be made. If one neglects contributions from the continuum, anomalous dimensions, and the gluon and four-quark condensates in Eq. (2.57), the following simple result is obtained:

$$M_N = -\frac{8\pi^2}{M^2}\langle\overline{q}q\rangle_{\mathrm{vac}} \ , \tag{2.58}$$

which is to be evaluated for $M^2 \sim 1\,\mathrm{GeV}^2$. This is known as the Ioffe formula, which we generalize to finite density in Sec. III F.

In Fig. 1 we plot the nucleon mass predicted by the ratio sum rule [Eq. (2.57)] as a function of Borel M^2 for several different continuum thresholds. We also plot the higher-order sum rule of Ioffe and Smilga [68]. We expect that the sum rule should be valid in a region near $M^2 = 1\,\mathrm{GeV}^2$ [6]. That is to say, the overlap between the region where the sum rule is dominated by the nucleon contribution and the region where the truncated operator product expansion is reliable is around $1\,\mathrm{GeV}^2$.

FIGURES

FIG. 1. Ratio sum-rule predictions for the nucleon mass M_N at zero density. The lower curves are from the ratio sum rule up to dimension-six condensates only [Eq. (2.57)], and the upper group of curves show the predictions of the sum rule from Ref. [68], which includes higher-dimensional operators. Anomalous-dimension corrections are neglected. In both cases, the three curves correspond to continuum thresholds $s_0 = 2.0\,\mathrm{GeV}^2$ (solid), $s_0 = 2.5\,\mathrm{GeV}^2$ (dashes), and $s_0 = 3.0\,\mathrm{GeV}^2$ (dot-dashes).

The sum rules for the nucleon are less than ideal for a number of reasons. First, we expect that the ratio sum-rule prediction for the nucleon, if well-satisfied, should have a flat region as a function of the Borel mass, which is just an auxiliary parameter. In fact, the nucleon sum rule is not close to flat if we truncate the OPE at dimension-six condensates, as we will do at finite density. On the other hand, the sum rule of Ioffe and Smilga [68], which includes various higher-dimensional condensates, becomes considerably flatter over a larger region. We interpret the change in the ratio within the fiducial region as a measure of the theoretical error bar for the mass. Second, perturbative corrections to many of the Wilson coefficients are large, although the net effect on the sum-rule predictions seems to be small [70]. Third, the continuum contribution is unavoidably large because the gap between the nucleon and higher-mass states is only of order 500 MeV. This is generally undesirable, because we only use a simple model to account for the continuum states; however, lattice tests [71] find the approximation to be quite reasonable (see Sec. VI B). Finally, it has been suggested that instanton effects are also important for the nucleon sum rules [72,73] and that inclusion of instanton effects help to stabilize the sum rule.

The vacuum sum rules for the nucleon (and other baryons) appear to be phenomeno-

logically successful [10]. However, there are a variety of ways in which the sum rules could be imprecise (or even fail). While this observation might make one hesitate before including the further complications of finite density, we think that the situation is actually quite favorable. Indeed, since we focus on *changes* in spectral properties with density, we are less sensitive to details that affect the *absolute* predictions of vacuum properties. We will rely on the cancellation of systematic discrepancies (including those from the truncation to dimension-six operators) by normalizing all finite-density self-energies to the zero-density prediction for the mass [18]. Moreover, even predictions with large uncertainties (*e.g.,* 50%) will be useful in assessing relativistic phenomenology. So we will proceed to generalize the nucleon sum-rule formalism, assuming that the zero-density limit is valid.

QCD sum-rule methods have also been widely applied to the calculation of baryons other than the nucleon and to meson masses, coupling constants, and form factors. Results for almost every low-spin ($J < 3$) meson have been calculated with only a handful of parameters. These calculations are well documented elsewhere; the book by Shifman [10] and the reviews by Reinders *et al.* [8] and Narison [9] provide good guides to the literature.

III. NUCLEONS AT FINITE DENSITY

A. Overview

In this section, we extend the sum-rule approach to the finite-density problem of nucleons propagating in infinite nuclear matter. We start with a review of relativistic phenomenology, which identifies the quantities to calculate and the physics we hope to test with the sum rules. Next we focus on changes in the chiral condensate at finite density, which suggests that nucleon propagation should be strongly modified. Then we put together these inputs through a generalization of the sum-rule approach, which encompasses several subsections. Finally we present results and mention alternative calculations. We have omitted some technical details in places so that we can focus on the physics, but complete details are available in the cited references.

B. Relativistic Nuclear Phenomenology

In this section, we review some elements of relativistic nuclear physics phenomenology. Our discussion follows that of Ref. [18]. Our goal is twofold: to explain the basic physics underlying the approach and to give insight into the appropriate way to model the spectral density for QCD sum rules at finite density. We also will introduce notation that will be used for the sum rules.

The most significant success of relativistic nuclear physics has been the economical description of proton-nucleus spin observables at intermediate energies for a wide range of target nuclei. This problem has been studied both purely phenomenologically using a global parameterization of the scattering data [31–33,38] and in a meson-theoretical framework based on a relativistic impulse approximation [34–37]. The key to the successes of both

approaches is that nucleon propagation in the nuclear medium is described by a Dirac equation featuring large Lorentz scalar and vector optical potentials. That is, the nucleon wave function ψ satisfies

$$(E\gamma_0 - \gamma \cdot \mathbf{q} - M_N - U)\psi = 0 \ , \qquad\qquad U \simeq S + V\gamma_0 \ , \tag{3.1}$$

where S and V are the scalar and vector optical potentials, M_N is the nucleon mass, and E is the nucleon energy.

Although the two approaches give slightly different optical potentials, the qualitative features are quite similar [31–38]:

- Attractive scalar (Re $S < 0$) and repulsive vector (Re $V > 0$) potentials are found, with magnitudes reaching several hundred MeV at nuclear matter saturation density.
- Significant cancellation between the potentials occurs, so that the effective nonrelativistic central potential is only tens of MeV in magnitude.
- The scalar and vector imaginary parts also exhibit significant cancellation; the imaginary parts are significantly smaller than the real parts.
- The real parts of the potentials have relatively weak energy dependence.

These qualities naturally suggest that a nucleon at intermediate energies can be regarded as a quasiparticle with large scalar and vector self-energies (corresponding to the optical potentials). The imaginary parts of the optical potential indicate that the width in energy of the quasinucleon excitation is relatively small on hadronic scales; that is, small compared to the spacing between the free-space nucleon and the Roper resonance. As discussed below, a similar picture emerges for nucleons below the Fermi sea in mean-field QHD [39,40] or relativistic Brueckner calculations [39,74–76].

This picture is different from conventional descriptions based on nonrelativistic NN potentials. It is worth asking why one might expect such a picture. The empirical low-energy NN scattering amplitudes are conventionally parameterized using Galilean invariants; one might, instead, choose to express these amplitudes using Lorentz invariants. In this case, one finds Lorentz scalar and vector components that are much larger than the amplitudes deduced from the nonrelativistic decomposition. In ordinary, spin-saturated nuclear matter, all of the other Lorentz components arising from the NN interaction largely average out, leaving only small effects; these include terms arising from one-pion exchange. Thus, the dynamics of neutral scalar and vector components are the most important for describing nucleons in bulk nuclear matter [39].

Relativistic hadronic field theories of nuclear phenomena such as quantum hadrodynamics (QHD) provide a qualitative description of this physics. In this description, large scalar and vector self-energies emerge at the mean-field level from the interaction of a nucleon with all other nucleons in the Fermi sea via the exchange of isoscalar scalar and vector mesons. This simple picture has many phenomenological successes; for example, relativistic mean-field models provide a quantitatively accurate description of many bulk properties of nuclei [39].

It is important to stress that the QCD sum rules we construct will not test this underlying dynamical picture of meson exchange as the origin of the quasinucleon self-energies. Concerns about issues such as the use of the Dirac equation to describe composite nucleons (such as the role of Z-graph physics) have dominated the past discussion of connections between relativistic nuclear physics and QCD [41–46,51,48]. We will briefly address some of these issues at the end of this article, but they do not affect our discussion. Instead, we consider the *spectral* properties themselves, which can be studied outside the context of a hadronic model. Having done such a study, one can compare to the predictions of Dirac phenomenology or QHD. The critical questions are the magnitudes and signs of the Lorentz scalar and vector parts of the optical potential.

To put this more concretely, we consider the nucleon propagator in a QHD theory

$$G(q) = -i \int d^4x \, e^{iq\cdot x} \, \langle \Psi_0 | T\psi(x)\overline{\psi}(0) | \Psi_0 \rangle \ , \tag{3.2}$$

where $|\Psi_0\rangle$ is the nuclear matter ground state and $\psi(x)$ is a nucleon field [39]. The analytic structure of G in the mean-field (and more sophisticated) approximations can suggest what we should expect to find for an analogous QCD correlator. The nucleon self-energy Σ can be defined formally in terms of the solution of Dyson's equation for the inverse propagator:

$$[G(q)]^{-1} = \not{q} - M_N - \Sigma(q) \ . \tag{3.3}$$

The self-energy can be identified directly from the analytic properties of the propagator $G(q)$. In particular, the discontinuities of G across the real q_0 axis, which are proportional to the spectral densities, can be used to extract the on-shell self-energy.

Since Lorentz transformation properties are at the heart of this problem, it is useful to identify the various Lorentz vectors in the problem. We will use these vectors to classify the possible terms in $G(q)$. There are only two independent vectors: q_μ itself and u_μ, the four-velocity of the matter. Since u_μ will play an essential role in what follows, it is useful to give some feeling for what u_μ means. The key point is that infinite nuclear matter has a natural rest frame specified by u_μ: the frame in which the expectation value of the baryon current is zero for all spatial components; the time component of the current in this frame is simply the baryon density ρ_N. In the rest frame of nuclear matter, the components of the four-velocity are $(1, \mathbf{0})$. An alternative way to understand u_μ is to consider infinite nuclear matter as the limit of a finite nucleus when the mass and particle number go to infinity. The four-velocity is simply given by $u_\mu = p_\mu/M$ where p_μ is the four-momentum of the nucleus and $M = \sqrt{p\cdot p}$ is the invariant mass. (In general, any four-vector thermodynamic quantity in an equilibrium system, such as the momentum or baryon densities, is proportional to u_μ.)

We start with a general decomposition of $G(q)$:

$$G(q) = G_s(q^2, q\cdot u) + G_q(q^2, q\cdot u)\not{q} + G_u(q^2, q\cdot u)\not{u} \ . \tag{3.4}$$

This form is determined by Lorentz covariance and the assumed invariance of the ground state under parity and time-reversal symmetries [77]. The self-energy can be decomposed similarly; this will define the notation used in subsequent sections. The nucleon self-energy is written as

$$\Sigma(q) = \widetilde{\Sigma}_s(q^2, q \cdot u) + \widetilde{\Sigma}_v^\mu(q)\gamma_\mu \ , \tag{3.5}$$

where

$$\widetilde{\Sigma}_v^\mu(q) = \Sigma_u(q^2, q \cdot u)u^\mu + \Sigma_q(q^2, q \cdot u)q^\mu \ . \tag{3.6}$$

We also define an in-medium scalar self-energy

$$\Sigma_s \equiv M_N^* - M_N \ , \quad M_N^* \equiv \frac{M_N + \widetilde{\Sigma}_s}{1 - \Sigma_q} \ , \tag{3.7}$$

and an in-medium vector self-energy

$$\Sigma_v \equiv \frac{\Sigma_u}{1 - \Sigma_q} \ , \tag{3.8}$$

The combinations in Eqs. (3.7) and (3.8) appear naturally when one solves for the nucleon pole. In a certain sense, the "scalar self-energy" of the nucleon in the medium is M_N^*; this is the scalar quantity we will calculate using the sum rules. However, we will follow the conventional nuclear physics nomenclature and refer to Σ_s as the scalar self-energy in the medium.

In the mean-field approximation, Σ_s and Σ_v are found to be real and independent of momentum, and Σ_q is taken to be identically zero. In this description, nucleons of any three-momentum appear as stable quasiparticles with self-energies that are nearly linear in the density up to nuclear matter density [39]. For mean-field models that provide quantitative fits to bulk properties of finite nuclei, the self-energies are typically several hundred MeV in magnitude at nuclear matter saturation density: $\Sigma_s \sim -350$ MeV and $\Sigma_v \sim +300$ MeV. These correspond to real, energy-independent optical potentials, S and V, and are similar in magnitude to the real parts of the optical potentials used to describe proton-nucleus scattering [31–38]. We wish to stress that the effective nucleon mass M_N^* defined in Eq. (3.7) is *not* equivalent to the usual nonrelativistic effective nucleon mass that is connected with the momentum dependence of the optical potential [78]. It comes from different physics and does not reduce to the nonrelativistic M_N^* in the nonrelativistic reduction of the model. We refer the reader to Ref. [79] for further discussion of the relations between different effective masses.

Clearly, mean-field physics is not the entire story. We know *a priori* that the optical potentials are complex and energy dependent. Thus, although mean-field models successfully describe a wide range of phenomena, their simplicity leads us to question whether the basic physics survives in a more sophisticated analysis. The most sophisticated relativistic calculations of nuclear matter have been performed in what is usually known as the Dirac Brueckner Hartree-Fock (DBHF) approximation [39,74–76]. This approximation incorporates effects from short-range correlations, which are critical in the nonrelativistic description of nuclear matter saturation. While these calculations involve some untested assumptions, they provide a unified and quantitative description of NN scattering observables and nuclear matter saturation properties.

Dirac Brueckner calculations generally find that the on-shell self-energies are only weakly dependent on the three-momentum $\mathbf{q}$. (This corresponds to a weak energy dependence for

the real parts of the relativistic scalar and vector optical potentials seen by a scattered nucleon.) Here, "on-shell" means that the self-energies are evaluated at the q_0 corresponding to the pole position, which is found by solving a transcendental equation for the self-consistent single-particle energy [39]. The self-energies Σ_s and Σ_v are found to be essentially similar in magnitude, sign, and density dependence to those from mean-field calculations. Furthermore, the magnitude of the dimensionless Σ_q, which is zero in the mean-field approximation, is typically much less than one in DBHF calculations (see, however, Ref. [75]). Thus the mean-field quasiparticle picture is qualitatively unchanged in the DBHF approximation. We use this picture to guide us in formulating our QCD sum-rule spectral ansatz.

In the mean-field approximation, the propagator with real self-energies in the rest frame of nuclear matter is

$$G(q) = \frac{1}{\not{q} - M_N - \Sigma(q)} \longrightarrow \lambda^2 \frac{\not{q} + M_N^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \overline{E}_q)} \ , \tag{3.9}$$

where E_q and $\overline{E}_q$, the positions of the positive- and negative-energy poles, are defined in Eqs. (3.10) and (3.11) below. We have introduced a common residue factor λ^2, which is unity here, but includes the factor $(1 - \Sigma_q)^{-1}$ in more general approximations. (Infinitesimals are not needed in the present discussion, so we suppress them.) Although, we have given the result in a particular frame, it is straightforward to go to any other frame via a Lorentz boost.

We can pick out the functions G_q, G_s, and G_u directly from Eq. (3.9) using Eq. (3.4). A Lehmann representation, obtained by inserting a complete set of intermediate states between the ψ and $\overline{\psi}$ in Eq. (3.2), shows that G_q, G_s, and G_u have the same singularity structure. In general, the entire real q_0 axis is cut (except for a small gap at the chemical potential), but in the mean-field approximation there are only two simple poles. The discontinuities of the G_i's across the real q_0 axis are proportional to the spectral functions. There are no singularities elsewhere in the complex q_0 plane.

In the mean-field approximation, the positive- and negative-energy poles in q_0 are at

$$E_q = \Sigma_v + \sqrt{\mathbf{q}^2 + {M_N^*}^2} \equiv \Sigma_v + E_q^* \ , \tag{3.10}$$

$$\overline{E}_q = \Sigma_v - \sqrt{\mathbf{q}^2 + {M_N^*}^2} \equiv \Sigma_v - E_q^* \tag{3.11}$$

The discontinuities of the propagator functions across the real q_0 axis, for real, fixed $|\mathbf{q}|$, are simply δ functions in this approximation:

$$\Delta G_s(q_0) = -2\pi i \frac{M_N^* \lambda^2}{2E_q^*} [\delta(q_0 - E_q) - \delta(q_0 - \overline{E}_q)] \ , \tag{3.12}$$

$$\Delta G_q(q_0) = -2\pi i \frac{\lambda^2}{2E_q^*} [\delta(q_0 - E_q) - \delta(q_0 - \overline{E}_q)] \ , \tag{3.13}$$

$$\Delta G_u(q_0) = +2\pi i \frac{\Sigma_v \lambda^2}{2E_q^*} [\delta(q_0 - E_q) - \delta(q_0 - \overline{E}_q)] \ . \tag{3.14}$$

These define the mean-field spectral densities up to an overall factor of $2\pi i$. It is clear from Eq. (3.9) that the *relative* residues of the $\not{q}$, scalar, and $\not{u}$ poles directly determine M_N^* and

Σ_v. These are the two independent quantities we wish to extract; when we generalize to the QCD case, these will be the quantities we want to extract via our spectral ansatz for the QCD nucleon correlator. Note, however, that in the mean-field approximation, the on-shell self-energies are independent of $\mathbf{q}$, while the self-energies we extract from the sum rules will depend explicitly (but weakly) on $\mathbf{q}$.[7]

Evidently, to match the empirical fact that quasinucleon single-particle energies are mostly unchanged at nuclear matter density, we must find significant cancellation between Σ_s and Σ_v to keep E_q roughly constant. In contrast, the mean-field ansatz predicts a significant shift of the negative-energy "pole" position $\overline{E}_q$ with increasing density, since the scalar and vector self-energies are both attractive in this channel. In reality, we expect a broad distribution of strength rather than a narrow excitation, so the simple ansatz is far more realistic for the positive-energy quasinucleon than for the antinucleon. Thus we find that, although a simple real momentum-independent description of the self-energies yields a plausible description of the nucleon in nuclear matter, it does a rather poor job of describing antinucleons in nuclear matter. It should be noted that this spread in the antinucleon strength occurs rather naturally in Dirac phenomenologically. While the imaginary parts of the scalar and vector self-energies tend to cancel for the nucleon, they add for the antinucleon, implying a large imaginary self-energy and, hence, a large width. Thus, while the neglect of the imaginary part of the self-energy may be justified in descriptions of the nucleon, it is certainly not justified in descriptions of the antinucleon. Accordingly, it is important when using such an overly simple description to ensure that calculated quantities do not depend strongly on contributions coming from the antinucleons.

One might imagine that, given the large self-energies predicted in relativistic models, one could find clear experimental signatures. This is not the case. The *individual* self-energies are not directly observed in nuclei; only combinations of scalar and vector appear. Moreover, given the large cancellations between scalar and vector in many observables, one finds that the best evidence of a modified relativistic effective mass M_N^*, for example, is rather indirect, coming from fits to spin observables in intermediate-energy scattering.

QCD sum rules, however, are not able to predict directly such spin observables. As a result, in our QCD sum-rule studies, we adopt an indirect approach. Instead of attempting to make direct predictions of experimental observables, we will use QCD sum rules to predict the individual scalar and vector self-energies themselves. To do so, we consider the analytic structure of something like the nucleon propagator in QHD models. This leads us to the natural analog in QCD of the QHD propagator: a correlator of interpolating fields with nucleon quantum numbers.

Our aim is to study the QCD correlator at finite density. In many ways the problem resembles the simple QHD problem discussed in this section. For example, we can exploit the fact that the correlator can be decomposed as in Eq. (3.4). We will also use the Lehmann

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[7]The momentum dependence means that the residues of the positive- and negative-energy poles will differ. This is an important effect in some other formulations of finite-density nucleon sum rules [80]. Because we will explicitly suppress negative-energy contributions, it will not be an important consideration in our sum rules.

representation to relate the discontinuities in q_0 across the real axis to the spectral densities, which, in turn, determine the correlator everywhere in the complex q_0 plane. Phenomenologically, we will assume a quasinucleon model to describe the region of the cut corresponding to the energy of a nucleon in nuclear matter. That is, we will take Eq. (3.9) as our ansatz for the quasinucleon contribution to the correlator, although we will allow the self-energies to depend on the three-momentum $\mathbf{q}$. We stress that all of this can be done without making the assumptions that QHD and the mean-field approximation are valid. All we assume is that the nucleon in medium is well approximated as a reasonably well-defined quasiparticle.

Lorentz covariance and the assumed invariance of the nuclear matter ground state under time-reversal and parity symmetries constrain the form of the spectral functions for the nucleon propagator in QHD models. In particular, they imply that only Lorentz scalar and vector self-energies (*i.e.*, no pseudoscalar or tensor components) are associated with a quasiparticle pole [77]. These same constraints also apply to the QCD correlator of nucleon interpolating fields. They *do not* involve further assumptions about hadronic degrees of freedom or other aspects of relativistic phenomenology. Therefore, the principal issue we address here is not whether there are scalar and vector self-energies that characterize the nucleon-like excitation in medium; this is a given once we believe that a quasiparticle approximation is reasonable. The real question is: What are the magnitudes, signs, and density dependencies of the self-energies? Phenomenologically, the quasiparticle energy of a nucleon in nuclear matter is only fractionally shifted from the free nucleon energy, and the excitation is only tens of MeV wide at most. This small shift will not be a built-in constraint, but should be predicted by the sum rules as well.

Of course the quasiparticle ansatz is overly simple: the true QCD correlator at finite density will not have simple poles on the real axis. However, we believe the width of the positive-energy quasinucleon excitation to be small on hadronic scales (and also compared to the energy over which we average in the sum rule), so we are justified in making a pole ansatz. Moreover, since this ansatz does not explicitly include a background or continuum at these energies, the effective self-energies we extract will account for *all* of the strength in the nuclear domain. In this sense our in-medium nucleon "pole" is like the ρ-meson "pole" in the vacuum sum-rules; it summarizes spectral strength over a narrow region. On the other hand, the spectral density on the negative-energy side, which corresponds to an antinucleon "pole," describes the spectral function quite badly; we expect a broad distribution as the density of the nuclear system is increased. Therefore, we will minimize our sensitivity to this part of the spectral density by constructing a sum rule that suppresses this contribution relative to the positive-energy side.

C. The Chiral Condensate in Nuclear Matter

Perhaps the most compelling evidence that light-quark hadron properties are significantly altered in matter stems from two basic facts: 1) The chiral condensate $\langle\overline{q}q\rangle$ plays an essential role in the structure of these hadrons, and 2) in the nuclear medium, the magnitude of the chiral condensate is substantially reduced relative to its vacuum value. This decrease of the chiral condensate in medium is often referred to as "partial restoration of chiral symmetry." In this section, we will review some of the important issues concerning the chiral condensate

at finite density. Additional details may be found in the recent review by Birse [50].

To leading order, the reduction in magnitude of the chiral condensate in nuclear matter from its vacuum value is simply the reduction per nucleon times the density of nucleons [12]. The essential observation is that the net amount a single isolated nucleon reduces the integrated scalar density is closely related to an experimental observable. In particular, the nucleon σ term, defined by

$$\sigma_N \equiv 2m_q \int d^3x \, (\langle \widetilde{N}|\overline{q}q|\widetilde{N}\rangle - \langle 0|\overline{q}q|0\rangle) \ , \tag{3.15}$$

with m_q the average of the up and down quark masses and $|\widetilde{N}\rangle$ a box-normalized nucleon state (see Sec. III E 2), is extractable from π-N scattering data after some (nontrivial) theoretical extrapolations.

Ignoring NN interactions, one can express the value of the chiral condensate at finite nuclear matter density as[8]

$$\frac{\langle\overline{q}q\rangle_{\rho_N}}{\langle\overline{q}q\rangle_{\rm vac}} = 1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} + \cdots \ , \tag{3.16}$$

where we have introduced the notation $\langle\widehat{O}_n\rangle_{\rho_N} \equiv \langle\Psi_0|\widehat{O}_n|\Psi_0\rangle$ and used the Gell-Mann–Oakes-Renner (GMOR) relation [81],

$$2m_q\langle\overline{q}q\rangle_{\rm vac} = -m_\pi^2 f_\pi^2 \ . \tag{3.17}$$

One can use Eq. (3.16) to get a first approximation to how much one expects the chiral condensate to change in the nuclear medium. For definiteness, we take nuclear matter saturation density to be $\rho_N = (110\,\mathrm{MeV})^3 \simeq 0.17\,\mathrm{fm}^{-3}$. The extraction of the σ term requires a sophisticated analysis and there remain significant uncertainties. A recent analysis gives a value for σ_N of approximately 45 MeV with an uncertainty in the 7–10 MeV range [82]; future experiments and theoretical work should tighten the limits further. Taking $\sigma_N = 45\,\mathrm{MeV}$ and using Eq. (3.16) gives $\langle\overline{q}q\rangle_{\rho_N}/\langle\overline{q}q\rangle_{\rm vac} \simeq 0.64$. Thus, this simple analysis suggests that the chiral condensate at nuclear matter density is substantially reduced from its free-space value.

The expression in Eq. (3.16) is, in a very real sense, model *independent* [51]. It is valid for any description of nuclear matter so long as the density is sufficiently low that one can ignore the effects of interactions. There has been considerable discussion [12,17,51,83–88,50] about corrections to Eq. (3.16), which are model *dependent*. Fortunately, there is good reason to believe that these corrections are small ($\leq 20\%$) up to nuclear matter density [51].

There is a very general way to relate the in-medium chiral condensate to the quark-mass dependence of the nuclear matter energy density. As shown in Ref. [51], one can use the

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[8]The condensate $\langle\overline{q}q\rangle$ is defined with an implicit normalization scale, typically taken to be 0.5–1.0 GeV. The ratio in Eq. (3.16), however, is renormalization-group invariant.

Hellmann-Feynman theorem [89] to express the in-medium condensate as[9]

$$\frac{\langle\overline{q}q\rangle_{\rho_N}}{\langle\overline{q}q\rangle_{\rm vac}} = 1 - \frac{1}{m_\pi^2 f_\pi^2}\left(\sigma_N\rho_N + m_q\frac{d\delta\mathcal{E}}{dm_q}\right) \ , \tag{3.18}$$

where $\delta\mathcal{E}$, which is of higher order in ρ_N, is the contribution to the energy density from interactions and Fermi motion.

The derivation of Eq. (3.18) is quite simple. The Hellmann-Feynman theorem [89] states that, given a Hermitian operator $H(\lambda)$, depending on the real parameter λ, with a set of normalized eigenstates $|\psi_i(\lambda)\rangle$, one obtains

$$\langle\psi_i(\lambda)|\frac{d}{d\lambda}H(\lambda)|\psi_i(\lambda)\rangle = \frac{d}{d\lambda}\langle\psi_i(\lambda)|H(\lambda)|\psi_i(\lambda)\rangle \ . \tag{3.19}$$

Now consider the application of this theorem to QCD. The operator H can be taken to be the QCD Hamiltonian,

$$H_{\rm QCD} = H^\chi_{\rm QCD} + \int d^3x\, m_q(\overline{u}u + \overline{d}d) + \tfrac{1}{2}\int d^3x\,(m_u - m_d)(\overline{u}u - \overline{d}d) + H^{\rm heavy}_{\rm QCD} \ , \tag{3.20}$$

where $H^\chi_{\rm QCD}$ is the QCD Hamiltonian in the chiral limit, $H^{\rm heavy}_{\rm QCD}$ contains the quark mass terms for strange and heavier quarks, and m_q is the average of the up and down quark masses. One can apply the Hellmann-Feynman theorem to this system with m_q playing the role of λ and $|\Psi_0\rangle$ and $|0\rangle$ playing the role of $|\psi_i(\lambda)\rangle$. After using the fact that the system is translationally invariant to remove an overall volume factor, one obtains

$$2m_q(\langle\overline{q}q\rangle_{\rho_N} - \langle\overline{q}q\rangle_{\rm vac}) = m_q\frac{d\mathcal{E}}{dm_q} \ , \tag{3.21}$$

where $\mathcal{E} = M_N\rho_N + \delta\mathcal{E}$ is the nuclear matter energy density.

Upon using this expression for the energy density, Eq. (3.21) becomes

$$2m_q(\langle\overline{q}q\rangle_{\rho_N} - \langle\overline{q}q\rangle_{\rm vac}) = m_q\left(\frac{dM_N}{dm_q}\rho_N + \frac{d\delta\mathcal{E}}{dm_q}\right) \ , \tag{3.22}$$

A second application of the Hellmann-Feynman theorem, this time using the nucleon and vacuum eigenstates of $H_{\rm QCD}$, yields $\sigma_N = m_q(dM_N/dm_q)$. This result plus the GMOR relation [Eq. (3.17)] gives us Eq. (3.18).

From Eq. (3.22), one might expect the corrections to Eq. (3.16) to be small at nuclear matter densities and below. The essential point is that the interaction energy density is much smaller than the energy density associated with the nucleon masses. (The interaction energy per nucleon is about 16 MeV, while the nucleon mass is nearly two orders of magnitude

[9] One can avoid renormalization subtleties in applying the Hellmann-Feynman theorem by working with bare quantities. Since Eq. (3.18) is renormalization-group invariant, the bare quantities can be replaced with analogous scale-dependent renormalized quantities.

larger.) Thus, one expects the first term on the right-hand side of Eq. (3.22) to dominate the second.

Of course, this argument is somewhat naive: $\delta\mathcal{E} \ll M_N\rho_N$ does not necessarily imply that $d\delta\mathcal{E}_\rho/dm_q \ll (dM_N/dm_q)\rho_N$. Nevertheless, the argument is quite suggestive. To proceed beyond this qualitative level, one needs to consider explicit models. The magnitude and even the sign of the correction to Eq. (3.16) have been subject to considerable recent discussion [12,17,51,83–88,50]. At present there are probably no models that can be used to calculate these corrections *reliably*. It is encouraging, however, to note that all of the models on the market, whether based on explicit quark degrees of freedom or hadronic degrees of freedom, give small corrections to the model-independent relation at least up to nuclear matter saturation density [50]. We are, therefore, reasonably confident that the in-medium condensate is reduced from its free space value by a substantial amount $\sim 35\%$.

However, one must ask whether the in-medium chiral condensate is a dynamically important quantity. This question was raised by Ericson [88] in the following context: The chiral condensate is a spatially averaged quantity. In any realistic description of nuclear matter there are spatial correlations in the local value of $\overline{q}q$; one might expect that inside nucleons and in their immediate neighborhood the condensate is very different from its vacuum value, while between nucleons the value of $\overline{q}q$ may not be much altered from the vacuum value. Given this picture, it is reasonable to ask whether this average quantity should play any special role. The condensate apparently represents the average of two very different kinds of physics, and one might imagine that the average could change substantially by simply increasing the number density of regions where $\overline{q}q$ differs from the vacuum value without anything special happening elsewhere.

As noted by Birse [50], however, this picture is not realistic: the range of the spatial correlations for $\overline{q}q$ is fairly long; it is typically given by two-pion exchange. Thus, at nuclear densities, any given nucleon is influenced by the change in the condensate induced by its neighbors, since the range (and the physical origin) is similar to that of the scalar meson of one-boson-exchange phenomenology. Moreover, if the picture of localized bits of altered condensate were correct and the chiral condensate itself were of no dynamical significance, as suggested by Ericson, then one could have dense matter with the condensate going to zero and eventually changing signs without anything special happening. As noted recently [90], however, this is not possible; the chiral condensate can be shown to be negative semi-definite.

What are the consequences of a decreasing chiral condensate? As stressed earlier, the mass of the nucleon is strongly tied to chiral symmetry breaking. This relationship is seen both in simple chiral models [91] and in Ioffe's QCD sum-rule treatment of the nucleon [6]. The reduction of the chiral condensate by about 35% in the nuclear medium suggests, but does not prove, that the nucleon mass (more precisely, its scalar self-energy) in the medium should be significantly reduced. To be consistent with the empirical fact that nucleons are very weakly bound in energy, there must also be a strong vector repulsion. This is the picture suggested by the Dirac phenomenology. We next turn to the formulation of QCD sum rules for nucleons in medium to see whether this qualitative picture plays out.

D. Formalism: Nucleons in Medium

1. Finite-Density Correlator

The generalization of the vacuum QCD sum rules to finite density starts with the same correlation function of interpolating fields as in Eq. (2.13). The only immediate difference is that, instead of a vacuum expectation value, matrix elements are evaluated in the finite-density ground state. Thus we focus on[10]

$$\Pi(q) \equiv i \int d^4x \, e^{iq\cdot x} \langle \Psi_0 | T\eta(x)\overline{\eta}(0) | \Psi_0 \rangle \ , \tag{3.23}$$

where $\eta(x)$ is a colorless interpolating field made up of quark fields with the quantum numbers of a nucleon. The ground state of nuclear matter $|\Psi_0\rangle$ is characterized by the rest frame nucleon density ρ_N and by the four-velocity u^μ; it is assumed to be invariant under parity and time reversal. Formally, we work at fixed volume and baryon number until the end, when we take the thermodynamic limit.

We can choose to work either at fixed density, with interpolating fields in the Heisenberg picture as at zero density, or with a chemical potential and grand canonical Heisenberg operators. Since we work exclusively at zero temperature here, the distinction between working with the density and with a chemical potential is not critical. However, there are several reasons to work in the Heisenberg picture:

1. We wish to exploit the zero-density sum rules as a means of normalizing the finite-density sum rules, so we want a smooth limit to zero density.

2. Our estimates for the finite-density matrix elements (condensates) will give them as functions of density, not chemical potential.

3. The physics of the chemical potential will not be reproduced by the sum rules. That is, the singularity structure signaled by the chemical potential will not be reflected on the OPE side; more simply, the sum rules don't "know" about the Fermi surface.

We generalize the discussion of Sec. II E by considering all nucleon interpolating fields (currents) that contain no derivatives and couple to spin-$\frac{1}{2}$ and isospin-$\frac{1}{2}$ states only. There are two linearly independent fields with these features corresponding to a scalar or pseudoscalar up-down diquark coupled to the remaining quark. For the proton these two independent interpolating fields are

$$\eta_1(x) = \epsilon_{abc}[u_a^T(x) C\gamma_5 d_b(x)] u_c(x) \ , \tag{3.24}$$

$$\eta_2(x) = \epsilon_{abc}[u_a^T(x) C d_b(x)] \gamma_5 u_c(x) \ , \tag{3.25}$$

where T denotes a transpose in Dirac space and C is the charge conjugation-matrix. The analogous fields for the neutron follow by interchanging the up and down quark fields and changing the overall sign.

---

[10] As explained later, it is not important whether we start with the time-ordered or retarded correlator.

Here we define a linear combination of these two fields:

$$\eta(x) = 2\left[t\eta_1(x) + \eta_2(x)\right], \tag{3.26}$$

where t is an arbitrary real parameter. The current with $t = -1$ corresponds to Ioffe's choice [6], which was considered in the discussion of the vacuum sum rule. If the correlator were calculated to arbitrary accuracy from both the QCD expansion and the phenomenological description, one would expect that the sum-rule predictions of physical observables would be independent of the choice of t. In practice, however, the QCD expansion is truncated and the phenomenological description is represented crudely. The criterion of choosing the interpolating field in QCD sum-rule applications is to maximize the coupling of the interpolating field to the desired physical intermediate state relative to other (continuum) states, while minimizing the contributions of higher-order terms in the OPE. These goals cannot be simultaneously realized. The optimal choice of the nucleon interpolating field seems to be close to Ioffe's choice. However, interpolating fields with $t \simeq -1.1$ have also been used in nucleon sum-rule studies [9], in particular, in studying direct small-scale instanton effects in nucleon sum rules [72,73]. We shall consider values in the range $-1.15 \leq t \leq -0.95$ here to evaluate the sensitivity of our predictions.

It is often remarked in the literature that one cannot work covariantly at finite density or temperature because of the existence of a preferred frame of reference, *i.e.*, the rest frame of nuclear matter. This is a misconception. While the ground state is not *invariant* under all Lorentz transformations (unlike the vacuum state), matrix elements in this state do have well-defined Lorentz transformation properties. So two observers in different frames can still compare calculations or observations as prescribed by special relativity. The new feature is the additional four-vector u_μ that must be transformed when making the comparisons and must be included when building tensors or identifying invariant functions. (The situation here is analogous to considering diagonal matrix elements of a spin-averaged proton state, which is also characterized by a single four-vector, the four-momentum of the proton.)

The correlation function $\Pi(q)$ is a 4×4 matrix in Dirac space, so we can expand it in the usual complete set of Dirac matrices. Using the transformation properties of $\eta(x)$ and keeping in mind the role of u_μ, we can constrain the form of $\Pi(q)$. The arguments are analogous to those in Chapter 16 of Ref. [63]. Lorentz covariance dictates that the general form of the correlator is

$$\Pi(q) = \Pi_s + \Pi_q \not{q} + \Pi_u \not{u} + \Pi_1 \gamma^5 + \Pi_2 \not{q}\gamma^5 + \Pi_3 \not{u}\gamma^5 + \Pi_4 (q_\mu u_\nu - q_\nu u_\mu)\sigma^{\mu\nu} + \Pi_5 \epsilon_{\mu\nu\kappa\lambda} q^\kappa u^\lambda \sigma^{\mu\nu}\ , \tag{3.27}$$

where the Π_i's are scalar functions of the invariants q^2 and $q{\cdot}u$. We assume that the nuclear matter ground state is invariant under parity and time reversal in its rest frame. In a general frame, we must take $u^\mu \to u_\mu$ as well as $q^\mu \to q_\mu$ under these transformations; thus q^2 and $q{\cdot}u$ are unchanged. The parity constraint implies $\Pi_1 = \Pi_2 = \Pi_3 = 0$. In the vacuum, a term proportional to $\sigma^{\mu\nu}$ can be excluded, because it can only be contracted with the symmetric combination $q_\mu q_\nu$. In finite-density nuclear matter, this argument is no longer sufficient, but the assumed parity and time-reversal invariance implies $\Pi_4 = \Pi_5 = 0$. (See Ref. [77] for a more complete discussion based on spectral functions.)

Thus Lorentz covariance, parity, and time reversal imply that $\Pi(q)$ has the form

$$\Pi(q) \equiv \Pi_s(q^2, q\cdot u) + \Pi_q(q^2, q\cdot u)\not{q} + \Pi_u(q^2, q\cdot u)\not{u} \ . \tag{3.28}$$

There are *three* distinct structures—scalar, $\not{q}$, and $\not{u}$—and thus three invariant functions of the two scalars q^2 and $q\cdot u$ (or any convenient combination). Recall that in the vacuum there are only two structures: scalar and $\not{q}$. In the zero-density limit, $\Pi_u \to 0$, and Π_s and Π_q become functions of q^2 alone. For simplicity, we specialize the sum rules to the rest frame of the nuclear medium, where the variables q_0 and $\mathbf{q}^2$ are most useful. A covariant form can be recovered in general by repeating the analysis with $q_0 \to q\cdot u$ and $-\mathbf{q}^2 \to \tilde{q}^2 \equiv q^2 - (q\cdot u)^2$.

We can project out the individual invariant functions by taking traces:

$$\Pi_s = \tfrac{1}{4}\mathrm{Tr}(\Pi) \ , \tag{3.29}$$

$$\Pi_q = \frac{1}{q^2 - (q\cdot u)^2}\Big[\tfrac{1}{4}\mathrm{Tr}(\not{q}\Pi) - \frac{q\cdot u}{4}\mathrm{Tr}(\not{u}\Pi)\Big] \ , \tag{3.30}$$

$$\Pi_u = \frac{1}{q^2 - (q\cdot u)^2}\Big[\frac{q^2}{4}\mathrm{Tr}(\not{u}\Pi) - \frac{q\cdot u}{4}\mathrm{Tr}(\not{q}\Pi)\Big] \ . \tag{3.31}$$

These projections require that $q^2 - (q\cdot u)^2$ be nonzero; this means $\mathbf{q} \neq 0$ in the rest frame. If this is not the case, there are only two functions, and the second is projected by a trace with γ^0.

2. Dispersion Relation at Fixed Three-Momentum

As discussed in Sec. II E, the fundamental physical objects in the sum rules are the spectral functions

$$\rho_{\alpha\beta}(q, u) = \frac{1}{2\pi}\int d^4x \ e^{iq\cdot x}\langle\Psi_0|\eta_\alpha(x)\overline{\eta}_\beta(0)|\Psi_0\rangle \ , \tag{3.32}$$

$$\tilde{\rho}_{\alpha\beta}(q, u) = \frac{1}{2\pi}\int d^4x \ e^{iq\cdot x}\langle\Psi_0|\overline{\eta}_\beta(0)\eta_\alpha(x)|\Psi_0\rangle \ . \tag{3.33}$$

Because the spectral functions are expectation values in an equilibrium state, they are characterized by the density in the rest frame of the medium and the medium four-velocity u^μ.[11] The sum rules consist of weighted integrals in energy of the spectral functions (with an analytic weighting function), obtained from the correlator through Borel transforms. All of the various correlators (time-ordered, advanced, retarded) have the same spectral functions, so it doesn't matter which correlator we start with.

The spectral functions can be written as a sum over a complete set of energy-momentum eigenstates. For example (recall that η is a Heisenberg picture operator),

---

[11]Relativistic thermodynamics dictates that expectation values in a uniform medium in equilibrium are characterized by a single four-velocity. Since all equilibrium four-vectors such as the baryon current or the four-momentum of the ground state will be proportional to u^μ, we could choose one of them as well. However, since in the end we wish to take the thermodynamic limit, it is most convenient to work with the four-velocity.

$$
\begin{aligned}
\rho_{\alpha\beta}(q,u) &= \frac{1}{2\pi}\sum_n \int d^4x \; e^{iq\cdot x}\langle\Psi_0|\eta_\alpha(x)|n\rangle\langle n|\overline{\eta}_\beta(0)|\Psi_0\rangle \\
&= \frac{1}{2\pi}\sum_n \int d^4x \; e^{iq\cdot x}\langle\Psi_0|e^{i\widehat{P}\cdot x}\eta_\alpha(0)e^{-i\widehat{P}\cdot x}|n\rangle\langle n|\overline{\eta}_\beta(0)|\Psi_0\rangle \\
&= \frac{1}{2\pi}\sum_n \int d^4x \; e^{i(q\cdot x+P_0\cdot x-P_n\cdot x)}\langle\Psi_0|\eta_\alpha(0)|n\rangle\langle n|\overline{\eta}_\beta(0)|\Psi_0\rangle \\
&= (2\pi)^3\sum_n \delta^{(4)}(q+P_0-P_n)\langle\Psi_0|\eta_\alpha(0)|n\rangle\langle n|\overline{\eta}_\beta(0)|\Psi_0\rangle \; . \qquad (3.34)
\end{aligned}
$$

Here, P_0^μ is the ground state four-momentum, equal to the product of the ground state mass with the four-velocity u^μ, and P_n^μ is the four-momentum of the state $|n\rangle$. Recall that we work at fixed volume and baryon number until the end, when the thermodynamic limit is to be taken.

At zero density, the spectral densities ρ and $\widetilde{\rho}$ (which include the nucleon and antinucleon states, respectively) are related by discrete space-time symmetries, because the vacuum is invariant under these operations [63]. In contrast, the finite-density ground state $|\Psi_0\rangle$ is not invariant under charge conjugation, so there is no definite relationship between the functions ρ and $\widetilde{\rho}$ and, in particular, the spectral densities for nucleon and antinucleon quasiparticles are not simply related.

The spectral representation of the correlator $\Pi(q)$ starts as an integral in energy over the spectral densities at fixed three-momentum. In the vacuum, where the invariant functions depend only on q^2, the separation of q_0 and $|\mathbf{q}|$ dependence is not necessary (or particularly useful). Also, the discrete symmetries require that the spectral density for negative-energy states (corresponding, for example, to the antiparticle) is equal (up to a sign) to the spectral density for positive-energy states. These properties allow us to change the integration over energy to one over q^2. In contrast, the four-velocity of the medium u_μ makes the distinction important in our case.

At finite density, we keep the dispersion relations as integrals over q_0, with the three-momentum $|\mathbf{q}|$ held fixed. This provides a clean identification of the intermediate quasi-nucleon states, which are naturally labeled by $|\mathbf{q}|$. The contribution from negative-energy quasinucleons (antinucleons) is clearly separated, which lets us isolate to a large degree the positive-energy quasinucleon contribution in the subsequent sum rule by adopting an appropriate weighting function.

We can write a dispersion relation for each of the Lorentz structures Π_i $[i=\{s,q,u\}]$,

$$
\Pi_i(q_0,|\mathbf{q}|) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega,|\mathbf{q}|)}{\omega-q_0} \; , \qquad (3.35)
$$

where we have omitted possible polynomials arising from subtractions, which will be eliminated by a subsequent Borel transform. We have also omitted the infinitesimals (which distinguish time-ordered and retarded correlators), since we will not evaluate q_0 on the real axis. The discontinuity, defined by

$$
\Delta\Pi_i(\omega,|\mathbf{q}|) \equiv \lim_{\epsilon\to 0^+}[\Pi_i(\omega+i\epsilon,|\mathbf{q}|)-\Pi_i(\omega-i\epsilon,|\mathbf{q}|)] \; , \qquad (3.36)
$$

contains the spectral information on the quasiparticle, quasihole, and higher-energy states. It is simply proportional to the sum of the spectral functions in Eqs. (3.32) and (3.33) [taking

into account the tensor structure]. The correlator is analytic except for cuts on the real q_0 axis (which vanish at the chemical potential) [92]. For q_0 off the real axis, one has

$$\Pi_i(q_0^*, |\mathbf{q}|) = [\Pi_i(q_0, |\mathbf{q}|)]^* \ , \tag{3.37}$$

which relates the function in the upper and lower half planes. The deep spacelike (Euclidean) limit with $\mathbf{q}$ fixed takes $q_0 \to i\infty$.

3. Phenomenological Ansatz

We generalize the usual zero-density ansatz for the spectral functions by assuming a quasiparticle pole for the nucleon,[12] with real self-energies (dependent on $\mathbf{q}$); all higher-energy excitations are included in a continuum contribution. We use the notation of Sec. III B. For infinite nuclear matter, where in the rest frame we can label a quasinucleon state by its three-momentum, energy dependence of optical potentials translates into three-momentum dependence of the self-energies. At the mean-field level, there is no dependence; all quasinucleons see the same self-energies. But even in the most sophisticated Dirac Brueckner Hartree-Fock (DBHF) calculations, the three-momentum dependence is weak (at least for bound and low-lying continuum states).

The quasiparticle-pole contribution to the correlator is

$$\Pi(q) \propto \frac{1}{(q^\mu - \widetilde{\Sigma}_v^\mu)\gamma_\mu - (M_{\rm N} + \widetilde{\Sigma}_s)} \ , \tag{3.38}$$

where $\widetilde{\Sigma}_v^\mu$ and $\widetilde{\Sigma}_s$ are the in-medium self-energies. In the language of the hadronic theories discussed in Sec. III B, these are the on-shell self-energies for a quasinucleon with three-momentum $\mathbf{q}$. The representations of the individual invariant functions are (in the nuclear matter rest frame)

$$\Pi_s(q_0, |\mathbf{q}|) = -\lambda_{\rm N}^{*\,2} \frac{M_{\rm N}^*}{(q_0 - E_q)(q_0 - \overline{E}_q)} + \cdots \ , \tag{3.39}$$

$$\Pi_q(q_0, |\mathbf{q}|) = -\lambda_{\rm N}^{*\,2} \frac{1}{(q_0 - E_q)(q_0 - \overline{E}_q)} + \cdots \ , \tag{3.40}$$

$$\Pi_u(q_0, |\mathbf{q}|) = \lambda_{\rm N}^{*\,2} \frac{\Sigma_v}{(q_0 - E_q)(q_0 - \overline{E}_q)} + \cdots \ , \tag{3.41}$$

where we have defined $M_{\rm N}^*$, Σ_v, E_q, and $\overline{E}_q$ as in Eqs. (3.7), (3.8), (3.10), and (3.11) and introduced an overall residue $\lambda_{\rm N}^{*\,2}$ that denotes the coupling of the interpolating field to the physical quasinucleon state. The positive- and negative-energy quasinucleon poles are

---

[12] Note that the sum rules do not distinguish between quasiparticles and quasiholes. The other nucleons only provide background fields to be seen by the injected quarks at relatively short times, so Pauli-principle correlations do not arise.

explicit, and $\cdots$ denotes the contribution from higher-energy states, which will be included later. Recall that we expect this to be a reasonable ansatz for representing the nucleon self-energy, but a poor ansatz for the antinucleon. Differences in the residues between positive- and negative-energy states are not important in our formalism, but can be critical in other analyses [80].

4. OPE at Fixed Three-Momentum

The next ingredient in our sum rule is an operator product expansion (OPE) of the time-ordered product in Eq. (3.23) at short distances. The correlator is studied in the limit that q_0 becomes large and imaginary while $|\mathbf{q}|$ remains fixed (in the nuclear matter rest frame). This limit takes $q^2 \to -\infty$ with $|q^2/q \cdot u| \to \infty$, which satisfies the conditions discussed in Ref. [30] for a short distance expansion.[13] It might appear that in this limit long-distance correlations in the spatial distance are possible. However, the singularities of the correlator lie on the light cone, so to the extent that we are dominated by these singularities (which are still in q^2 at finite density), we have short distance as well as short time in this limit. We also apply Borel transforms in q_0^2 (or, equivalently, in q^2 with fixed $\mathbf{q}^2$), which implies that only terms in the expansion that are discontinuous across the real q_0 axis contribute to the sum rules (for example, polynomials are eliminated).

At finite density, the OPE for the invariant functions of the nucleon correlator take the general form

$$\Pi_i(q^2, q \cdot u) = \sum_n C_n^i(q^2, q \cdot u) \langle \widehat{O}_n \rangle_{\rho_N} \ . \tag{3.42}$$

(Recall $\langle \widehat{O}_n \rangle_{\rho_N} \equiv \langle \Psi_0 | \widehat{O}_n | \Psi_0 \rangle$.) The $C_n^i(q^2, q \cdot u)$ $(i = \{s, q, u\})$ are the Wilson coefficients, which depend on QCD Lagrangian parameters such as the quark masses and the strong coupling constant. The Wilson coefficients in the OPE only depend on q^μ, and the ground-state expectation values of the operators are proportional to tensors constructed from the nuclear matter four-velocity u^μ, the metric $g^{\mu\nu}$, and the antisymmetric tensor $\epsilon^{\kappa\lambda\mu\nu}$. In Eq. (3.42) we incorporate the contraction of q^μ (from the OPE) and u^μ (from the ground-state expectation values of the operators) into the definition of the Wilson coefficients $C_n^i(q^2, q \cdot u)$. Thus the dependence on $q \cdot u$ is solely in the form of polynomial factors. We have suppressed the dependence on the normalization point μ.

The $\widehat{O}_n$ are local composite operators constructed from quark and gluon fields; examples of such operators are $\overline{q}q$ and $(\alpha_s/\pi)G^2$. The ground-state expectation values of these operators are the in-medium condensates. The operators are defined so that the density dependence of the correlator resides solely in the in-medium condensates; thus, the only

---

[13] Note that in Refs. [12,17,26] the finite-density correlator is studied using kinematics analogous to that of deep-inelastic scattering (*i.e.*, $q^2/q \cdot u$ is fixed and finite). This ensures that only the light cone is probed, but does not imply a short-distance expansion. Furthermore, the identification of physical quasinucleon intermediate states is obscured.

substantial difference from the vacuum calculations is that more composite operator matrix elements are nonzero in nuclear matter. The operators $\widehat{O}_n$ are ordered by dimension (measured as a power of mass) and the $C_n^i(q^2, q \cdot u)$ for higher-dimensional operators fall off by corresponding powers of $Q^2 \equiv -q^2$. Therefore, for sufficiently large Q^2, the operators of lowest dimension dominate, and the OPE can be truncated after a small number of lower-dimensional operators.

In the next subsections we consider in turn the calculation of the Wilson coefficients and the estimation of finite-density condensates.

5. Calculating Wilson Coefficients

We define the composite operators in the operator product expansion at finite density using the same renormalization prescription and subtractions as at zero density. This means that the coefficients of the OPE, the Wilson coefficients, will be independent of density. One can thus calculate the Wilson coefficients following the same methods developed for the vacuum sum rules (see, for example, Ref. [8]).

Note, however, that expectation values of local operators with any integer spin can be nonzero in medium due to the appearance of the additional four-vector u^μ. Consequently, there are a large number of new condensates, which, while vanishing in vacuum due to Lorentz invariance, will survive in medium. Thus one needs to calculate the Wilson coefficients of the operators corresponding to these new condensates.

Here we discuss some simple rules for the calculation of Wilson coefficients using the fixed-point gauge and background-field techniques, which have proven to be convenient for light-quark systems. We will assume that working to lowest order in perturbation theory is sufficient to calculate the Wilson coefficients with acceptable accuracy.

By construction, the hadron interpolating fields are colorless; thus the correlators of these interpolating fields are gauge invariant. Therefore, one can evaluate the correlators in any desired gauge. Here we adopt the fixed-point gauge, which was introduced for use in electrodynamics in Refs. [93,94], and reintroduced for use in QCD in Ref. [95]. The fixed-point gauge condition is

$$x_\mu \mathcal{A}^\mu(x) = 0 \ , \tag{3.43}$$

where $\mathcal{A}^\mu \equiv A^{A\mu} t^A$, with $A^{A\mu}$ the gluon field and $t^A \equiv \lambda^A/2$ the color SU(3) generators in the fundamental representation. In this gauge, the gluon field $\mathcal{A}_\nu$ can be expressed directly in terms of the gluon field tensor $\mathcal{G}_{\mu\nu}$ [95–99,8]:

$$\mathcal{A}_\nu(x) = \int_0^1 d\alpha\, \alpha x^\mu \mathcal{G}_{\mu\nu}(\alpha x) = \tfrac{1}{2} x^\mu \mathcal{G}_{\mu\nu}(0) + \tfrac{1}{3} x^\lambda x^\mu (D_\lambda \mathcal{G}_{\mu\nu})_{x=0} + \cdots \ , \tag{3.44}$$

where $\mathcal{G}_{\mu\nu} \equiv G^A_{\mu\nu} t^A \equiv D_\mu \mathcal{A}_\nu - D_\nu \mathcal{A}_\mu$, with $D_\mu \equiv \partial_\mu - i g_s \mathcal{A}_\mu$ the covariant derivative. This allows one to obtain manifestly gauge-invariant results in a relatively simple way.

In the background-field method [96–99,8], the presence of nonperturbative quark and gluon condensates is parameterized by Grassmann background quark fields, $\chi^q_{a\alpha}$ and $\overline{\chi}^q_{a\alpha}$, and a classical background gluon field $F^A_{\mu\nu}$. It is most convenient to first work in coordinate

space and then transform to momentum space. The coordinate-space quark propagator for light quarks in the presence of the background fields takes the following form in the fixed-point gauge [8]:

$$
\begin{aligned}
S^q_{ab,\alpha\beta}(x,0) &\equiv \langle T q_{a\alpha}(x)\overline{q}_{b\beta}(0)\rangle_{\rho_N} \\
&= \frac{i}{2\pi^2}\delta_{ab}\frac{1}{(x^2)^2}[\not{x}]_{\alpha\beta} - \frac{im_q}{4\pi^2}\delta_{ab}\frac{\delta_{\alpha\beta}}{x^2} \\
&\quad + \chi^q_{a\alpha}(x)\overline{\chi}^q_{b\beta}(0) - \frac{ig_s}{32\pi^2}F^A_{\mu\nu}(0)t^A_{ab}\frac{1}{x^2}[\not{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\not{x}]_{\alpha\beta} + \cdots \ ,
\end{aligned}
\tag{3.45}
$$

where the first and second terms are the expansion of the free quark propagator to first order in the quark mass, and the third and fourth terms are the contributions due to the background quark and gluon fields, respectively. The gluonic contribution to Eq. (3.45) comes from a single gluon insertion, retaining only the leading term in the short-distance expansion of the gluon field [see Eq. (3.44)]. Contributions from derivatives of the gluon field tensor, which are less singular at $x \to 0$, and additional gluon insertions are not included. These refinements are not needed given the level of truncation in the OPE considered in this review.

Products of Grassmann background quark fields and classical background gluon fields obtained in Eq. (3.45) correspond to ground-state matrix elements of the corresponding quark and gluon operators:

$$
\begin{aligned}
&\chi^q_{a\alpha}(x)\overline{\chi}^q_{b\beta}(0) = \langle q_{a\alpha}(x)\overline{q}_{b\beta}(0)\rangle_{\rho_N} \ , \qquad F^A_{\kappa\lambda}F^B_{\mu\nu} = \langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_{\rho_N} \ , \\
&\chi^q_{a\alpha}\overline{\chi}^q_{b\beta}F^A_{\mu\nu} = \langle q_{a\alpha}\overline{q}_{b\beta}G^A_{\mu\nu}\rangle_{\rho_N} \ , \qquad \chi^q_{a\alpha}\overline{\chi}^q_{b\beta}\chi^q_{c\gamma}\overline{\chi}^q_{d\delta} = \langle q_{a\alpha}\overline{q}_{b\beta}q_{c\gamma}\overline{q}_{d\delta}\rangle_{\rho_N} \ ,
\end{aligned}
\tag{3.46}
$$

where the fields are evaluated at $x = 0$ unless otherwise noted. In Eq. (3.46), we have only shown those matrix elements that are needed in order to carry out the OPE to the level we are considering. Thus we evaluate the fields in the higher-dimensional operators at the same point, since nonlocalities would only introduce condensates that are higher in dimension than those we wish to consider. The composite operators in Eq. (3.46) are implicitly normal-ordered with respect to the perturbative vacuum at zero density; we can write these matrix elements in terms of scalar local condensates by projecting out the Dirac, Lorentz, and color structure and performing a short-distance expansion if necessary. We discuss this procedure in detail; it is through this discussion that we introduce the relevant condensates for sum-rule calculations.

The Dirac and color structure of the matrix element $\langle q_{a\alpha}(x)\overline{q}_{b\beta}(0)\rangle_{\rho_N}$ can be projected out to obtain

$$
\langle q_{a\alpha}(x)\overline{q}_{b\beta}(0)\rangle_{\rho_N} = -\frac{\delta_{ab}}{12}\left[\langle\overline{q}(0)q(x)\rangle_{\rho_N}\delta_{\alpha\beta} + \langle\overline{q}(0)\gamma_\lambda q(x)\rangle_{\rho_N}\gamma^\lambda_{\alpha\beta}\right] \ ,
\tag{3.47}
$$

since nuclear matter is colorless and the ground state is (assumed to be) invariant under parity and time reversal. (Other matrix elements of the form $\langle\overline{q}(0)\Gamma q(x)\rangle_{\rho_N}$ do not contribute due to parity and/or time reversal.) We evaluate Eq. (3.47) at short distances by expanding the quark field $q(x)$ in a Taylor series:

$$q(x) = q(0) + x^\mu(\partial_\mu q)_{x=0} + \tfrac{1}{2}x^\mu x^\nu(\partial_\mu\partial_\nu q)_{x=0} + \cdots \ . \tag{3.48}$$

However, since the correlator is gauge invariant, the ordinary derivatives in Eq. (3.48) must ultimately become covariant derivatives. In standard calculations, gluon fields in higher-order terms of the OPE combine with the ordinary derivatives in lower-order terms to form covariant derivatives.

The situation is much more straightforward in the fixed-point gauge; the ordinary derivatives can be replaced with covariant derivatives immediately. We follow the discussion of Ref. [98]. Using the fixed-point gauge condition in Eq. (3.43), and expanding the gluon field, one obtains

$$x^\nu\mathcal{A}_\nu(0) + x^\mu x^\nu(\partial_\mu\mathcal{A}_\nu)_{x=0} + \tfrac{1}{2}x^\lambda x^\mu x^\nu(\partial_\lambda\partial_\mu\mathcal{A}_\nu)_{x=0} + \cdots = 0 \ . \tag{3.49}$$

Since x is arbitrary, the individual terms of Eq. (3.49) must vanish; using this fact, one can readily show

$$x^\mu(D_\mu q)_{x=0} = x^\mu(\partial_\mu q)_{x=0} \ , \qquad x^\mu x^\nu(D_\mu D_\nu q)_{x=0} = x^\mu x^\nu(\partial_\mu\partial_\nu q)_{x=0} \ , \tag{3.50}$$

and so on. Combining this result with Eq. (3.48), one derives the following covariant Taylor expansion:

$$q(x) = q(0) + x^\mu(D_\mu q)_{x=0} + \tfrac{1}{2}x^\mu x^\nu(D_\mu D_\nu q)_{x=0} + \cdots \ . \tag{3.51}$$

An analogous expansion of the gluon field tensor at short distances [used in Eq. (3.44)] is proved to all orders using mathematical induction in Ref. [99]. Therefore, we obtain

$$\begin{aligned}\langle q_{a\alpha}(x)\overline{q}_{b\beta}(0)\rangle_{\rho_N} = -\frac{\delta_{ab}}{12}[(&\langle\overline{q}q\rangle_{\rho_N} + x^\mu\langle\overline{q}D_\mu q\rangle_{\rho_N} + \tfrac{1}{2}x^\mu x^\nu\langle\overline{q}D_\mu D_\nu q\rangle_{\rho_N} + \cdots)\delta_{\alpha\beta} \\ &+ (\langle\overline{q}\gamma_\lambda q\rangle_{\rho_N} + x^\mu\langle\overline{q}\gamma_\lambda D_\mu q\rangle_{\rho_N} + \tfrac{1}{2}x^\mu x^\nu\langle\overline{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} + \cdots)\gamma^\lambda_{\alpha\beta}] \ , \end{aligned} \tag{3.52}$$

where all fields and field derivatives in the condensates are evaluated at $x = 0$.

It is useful to note one particular calculational convenience: Since the Dirac matrices involved in calculating the Wilson coefficient of $\langle\overline{q}D_{\mu_1}\cdots D_{\mu_n}q\rangle_{\rho_N}$ $[\langle\overline{q}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}q\rangle_{\rho_N}]$ are the same as those involved in calculating the Wilson coefficient of $\langle\overline{q}q\rangle_{\rho_N}$ $[\langle\overline{q}\gamma_\mu q\rangle_{\rho_N}]$, we conclude that the coordinate-space coefficients are related as follows:

$$C_{\overline{q}D_{\mu_1}\cdots D_{\mu_n}q}(x) = \frac{1}{n!}x^{\mu_1}\cdots x^{\mu_n}C_{\overline{q}q}(x) \ , \tag{3.53}$$

$$C_{\overline{q}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}q}(x) = \frac{1}{n!}x^{\mu_1}\cdots x^{\mu_n}C_{\overline{q}\gamma_\mu q}(x) \ . \tag{3.54}$$

This implies that the momentum-space Wilson coefficients are related by

$$C_{\overline{q}D_{\mu_1}\cdots D_{\mu_n}q}(q) = \frac{(-i)^n}{n!}\left(\frac{\partial}{\partial q_{\mu_1}}\cdots\frac{\partial}{\partial q_{\mu_n}}\right)C_{\overline{q}q}(q) \ , \tag{3.55}$$

$$C_{\overline{q}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}q}(q) = \frac{(-i)^n}{n!}\left(\frac{\partial}{\partial q_{\mu_1}}\cdots\frac{\partial}{\partial q_{\mu_n}}\right)C_{\overline{q}\gamma_\mu q}(q) \ . \tag{3.56}$$

We now proceed to evaluate the condensates appearing in Eq. (3.52) in terms of expectation values of scalar operators multiplied by quantities that contain the Lorentz structure of the original condensates. In vacuum, these condensates can only be expressed in terms of the metric tensor $g^{\mu\nu}$ and the antisymmetric tensor $\epsilon^{\kappa\lambda\mu\nu}$; thus condensates with an odd number of uncontracted Lorentz indices must vanish in the vacuum. In-medium condensates, however, can also be expressed in terms of the nuclear matter four-velocity u^μ, which leads to new condensates and new Lorentz structures. The general procedure for evaluating the condensates in Eq. (3.52) is to write each as a sum of all possible Lorentz structures with unknown coefficients. These coefficients, which will turn out to be expectation values of scalar operators, can then be determined by taking appropriate traces over the Lorentz indices. Following this procedure, one can express various condensates in Eq. (3.52) as [20,100]

$$\langle\overline{q}\gamma_\mu q\rangle_{\rho_N} = \langle\overline{q}\not{u}q\rangle_{\rho_N} u_\mu \ , \tag{3.57}$$

$$\langle\overline{q}D_\mu q\rangle_{\rho_N} = \langle\overline{q}u\cdot Dq\rangle_{\rho_N} u_\mu = -im_q\langle\overline{q}\not{u}q\rangle_{\rho_N} u_\mu \ , \tag{3.58}$$

$$\langle\overline{q}\gamma_\mu D_\nu q\rangle_{\rho_N} = \tfrac{4}{3}\langle\overline{q}\not{u}u\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \tfrac{1}{4}g_{\mu\nu}) + \frac{i}{3}m_q\langle\overline{q}q\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}) \ , \tag{3.59}$$

$$\langle\overline{q}D_\mu D_\nu q\rangle_{\rho_N} = \tfrac{4}{3}\langle\overline{q}u\cdot D\,u\cdot Dq\rangle_{\rho_N}(u_\mu u_\nu - \tfrac{1}{4}g_{\mu\nu}) - \tfrac{1}{6}\langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}(u_\mu u_\nu - g_{\mu\nu}) \ , \tag{3.60}$$

$$\begin{aligned}\langle\overline{q}\gamma_\lambda D_\mu D_\nu q\rangle_{\rho_N} &= 2\langle\overline{q}\not{u}u\cdot D\,u\cdot Dq\rangle_{\rho_N}[u_\lambda u_\mu u_\nu - \tfrac{1}{6}(u_\lambda g_{\mu\nu} + u_\mu g_{\lambda\nu} + u_\nu g_{\lambda\mu})] \\ &\quad - \tfrac{1}{6}\langle g_s\overline{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}(u_\lambda u_\mu u_\nu - u_\lambda g_{\mu\nu}) \ ,\end{aligned} \tag{3.61}$$

where the equations of motion have been used,[14] and $O(m_q^2)$ terms have been neglected. Thus the expansion of $\langle q_{a\alpha}(0)\overline{q}_{b\beta}(x)\rangle_{\rho_N}$ up to dimension five includes quark condensates and quark-gluon condensates.

Another source of quark-gluon condensates is from contributions of the form $\chi^q_{a\alpha}\overline{\chi}^q_{b\beta}F^A_{\mu\nu}$ in Eq. (3.45). The corresponding matrix element can be decomposed as

$$\begin{aligned}\langle g_s q_{a\alpha}\overline{q}_{b\beta}G^A_{\mu\nu}\rangle_{\rho_N} &= -\frac{t^A_{ab}}{96}\Big\{\langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}\Big[\sigma_{\mu\nu} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}\Big]_{\alpha\beta} \\ &+ \langle g_s\overline{q}\not{u}\sigma\cdot\mathcal{G}q\rangle_{\rho_N}\Big[\sigma_{\mu\nu}\not{u} + i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\Big]_{\alpha\beta} \\ &- 4\Big(\langle\overline{q}u\cdot D\,u\cdot Dq\rangle_{\rho_N} + im_q\langle\overline{q}\not{u}u\cdot Dq\rangle_{\rho_N}\Big)\Big[\sigma_{\mu\nu} + 2i(u_\mu\gamma_\nu - u_\nu\gamma_\mu)\not{u}\Big]_{\alpha\beta}\Big\} \ ,\end{aligned} \tag{3.62}$$

which is obtained by projecting out the color, Dirac, and Lorentz structure by taking appropriate traces. Many of the "condensates" encountered in the derivation of Eq. (3.62) vanish due to the assumed parity and time-reversal invariance of the nuclear matter ground state.

[14] To obtain the second equality in Eq. (3.58), we use the identity $D_\mu \equiv \frac{1}{2}(\gamma_\mu\not{D} + \not{D}\gamma_\mu)$ and translation invariance, which implies $\langle\overline{q}i\not{D}\not{u}q\rangle_{\rho_N} = -\langle\overline{q}i\overleftarrow{\not{D}}\not{u}q\rangle_{\rho_N}$.

The dimension-four gluon condensates arise from factors of $F^A_{\kappa\lambda}F^B_{\mu\nu}$ in Eq. (3.45).[15] The matrix element $\langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_{\rho_N}$ can be written in terms of two independent gluon condensates, which, in the rest frame, are proportional to $\langle \mathbf{E}^2 - \mathbf{B}^2\rangle_{\rho_N} = -\frac{1}{2}\langle G^2\rangle_{\rho_N}$ and $\langle \mathbf{E}^2 + \mathbf{B}^2\rangle_{\rho_N}$, where $\mathbf{E}^A$ and $\mathbf{B}^A$ are the color-electric and color-magnetic fields. Contributions from $\langle \mathbf{E}^2 + \mathbf{B}^2\rangle_{\rho_N}$ are small and are neglected in the sequel. Thus, we are left with

$$\langle G^A_{\kappa\lambda}G^B_{\mu\nu}\rangle_{\rho_N} = \frac{\delta^{AB}}{96}[\langle G^2\rangle_{\rho_N}(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu}) + O(\langle \mathbf{E}^2 + \mathbf{B}^2)\rangle_{\rho_N})] \; . \tag{3.63}$$

At dimension six we only consider the four-quark condensates. The leading-order four-quark condensate contributions to the baryon correlators arise at tree level; thus they do not carry the suppression factors associated with loops. Such contributions appear as terms proportional to $\chi^u_{a\alpha}\overline{\chi}^u_{b\beta}\chi^d_{c\gamma}\overline{\chi}^d_{d\delta}$, for example. The Lorentz, Dirac, and color structure of these matrix elements can be projected out in a manner similar to that discussed above. There are, in general, a large number of four-quark condensates contributing to the baryon correlators. If one assumes in-medium factorization (or ground-state saturation), these four-quark condensates can be expressed in terms of a few dimension-three quark condensates. The basic factorization formulas are [20,100]

$$\langle \overline{u}_{a\alpha}u_{b\beta}\overline{u}_{c\gamma}u_{d\delta}\rangle_{\rho_N} \simeq \langle \overline{u}_{a\alpha}u_{b\beta}\rangle_{\rho_N}\langle \overline{u}_{c\gamma}u_{d\delta}\rangle_{\rho_N} - \langle \overline{u}_{a\alpha}u_{d\delta}\rangle_{\rho_N}\langle \overline{u}_{c\gamma}u_{b\beta}\rangle_{\rho_N} \; , \tag{3.64}$$

$$\langle \overline{u}_{a\alpha}u_{b\beta}\;\overline{d}_{c\gamma}d_{d\delta}\rangle_{\rho_N} \simeq \langle \overline{u}_{a\alpha}u_{b\beta}\rangle_{\rho_N}\langle \overline{d}_{c\gamma}d_{d\delta}\rangle_{\rho_N} \; . \tag{3.65}$$

However, factorization has not been justified for nuclear matter and may well be wrong [18,22].

Summarizing the rules for calculating Wilson coefficients:

1. Apply Wick's theorem to the time-ordered product in the correlator, retaining only those contributions in which the quark fields are fully contracted;

2. use the background quark propagator [Eq. (3.45)], rather than the free quark propagator, for each contraction;

3. express the various condensates in terms of scalar operators multiplied by appropriate tensors.

In order to transform the expression to momentum space one can use the formulas [98,8]

---

[15] Here, in the calculation of the Wilson coefficients multiplying the gluon condensates, we neglect the quark masses. Note that the factor $F^A_{\kappa\lambda}F^B_{\mu\nu}$ comes from single gluon insertions in two of the quark propagators. One must also consider two gluon lines emanating from the same quark propagator, which gives contributions proportional to $\langle(\alpha_s/\pi)(\mathbf{E}^2 + \mathbf{B}^2)\rangle_{\rho_N}$ in the massless quark limit. Since this condensate only gives very small contributions to the baryon sum rules, for simplicity we omit these terms entirely. Note that the corresponding coefficients in Refs. [20] and [25] are missing the two-gluon-line contributions.

$$\int \frac{d^4x}{x^2} e^{iq\cdot x} = -\frac{4\pi^2 i}{q^2} \; , \tag{3.66}$$

$$\int \frac{d^4x}{(x^2)^n} e^{iq\cdot x} = \frac{i(-1)^n 2^{4-2n}\pi^2}{\Gamma(n-1)\Gamma(n)} (q^2)^{n-2} \ln(-q^2) + P_{n-2}(q^2) \qquad (n \geq 2) \; , \tag{3.67}$$

and their derivatives with respect to q^μ. $P_m(q^2)$ is a polynomial in q^2 of degree m with divergent coefficients. The precise forms of the polynomials are not important, since they do not contribute to the Borel transformed sum rules. Following this procedure, one gets the whole OPE expression for the correlator, from which one can easily identify the Wilson coefficients.

One can now apply these simple rules to evaluate the nucleon correlator. For convenience we separate the invariant functions into pieces that are even and odd in q_0:

$$\Pi_i(q_0, |\mathbf{q}|) = \Pi_i^{\mathrm{E}}(q_0^2, |\mathbf{q}|) + q_0 \Pi_i^{\mathrm{O}}(q_0^2, |\mathbf{q}|) \; . \tag{3.68}$$

The full results for the general interpolating field of Eq. (3.26) are given in Ref. [25]; here we list the dominant terms:

$$\begin{aligned} \Pi_s^{\mathrm{E}} = {} & \frac{c_1}{16\pi^2} q^2 \ln(-q^2) \langle \overline{q} q \rangle_{\rho_N} + \frac{3c_2}{16\pi^2} \ln(-q^2) \langle g_s \overline{q} \sigma \cdot \mathcal{G} q \rangle_{\rho_N} \\ & + \frac{2c_3}{3\pi^2} \frac{q_0^2}{q^2} (\langle \overline{q} i D_0 i D_0 q \rangle_{\rho_N} + \tfrac{1}{8} \langle g_s \overline{q} \sigma \cdot \mathcal{G} q \rangle_{\rho_N}) + \cdots \; , \end{aligned} \tag{3.69}$$

$$\Pi_s^{\mathrm{O}} = -\frac{c_1}{8\pi^2} \ln(-q^2) \langle \overline{q} i D_0 q \rangle_{\rho_N} - \frac{c_1}{3q^2} \langle \overline{q} q \rangle_{\rho_N} \langle q^\dagger q \rangle_{\rho_N} + \cdots \; , \tag{3.70}$$

$$\begin{aligned} \Pi_q^{\mathrm{E}} = {} & -\frac{c_4}{512\pi^4} (q^2)^2 \ln(-q^2) + \frac{c_4}{72\pi^2} \left(5 \ln(-q^2) - \frac{8q_0^2}{q^2} \right) \langle q^\dagger i D_0 q \rangle_{\rho_N} \\ & - \frac{c_4}{256\pi^2} \ln(-q^2) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho_N} - \frac{c_1}{6q^2} \langle \overline{q} q \rangle_{\rho_N}^2 - \frac{c_4}{6q^2} \langle q^\dagger q \rangle_{\rho_N}^2 + \cdots \; , \end{aligned} \tag{3.71}$$

$$\begin{aligned} \Pi_q^{\mathrm{O}} = {} & \frac{c_4}{24\pi^2} \ln(-q^2) \langle q^\dagger q \rangle_{\rho_N} + \frac{c_5}{72\pi^2 q^2} \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N} \\ & - \frac{c_4}{12\pi^2 q^2} \left(1 + \frac{2q_0^2}{q^2} \right) (\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N} + \tfrac{1}{12} \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N}) + \cdots \; , \end{aligned} \tag{3.72}$$

$$\begin{aligned} \Pi_u^{\mathrm{E}} = {} & \frac{c_4}{12\pi^2} q^2 \ln(-q^2) \langle q^\dagger q \rangle_{\rho_N} - \frac{c_5}{48\pi^2} \ln(-q^2) \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N} \\ & + \frac{c_4}{2\pi^2} \frac{q_0^2}{q^2} (\langle q^\dagger i D_0 i D_0 q \rangle_{\rho_N} + \tfrac{1}{12} \langle g_s q^\dagger \sigma \cdot \mathcal{G} q \rangle_{\rho_N}) + \cdots \; , \end{aligned} \tag{3.73}$$

$$\Pi_u^{\mathrm{O}} = -\frac{5c_4}{18\pi^2} \ln(-q^2) \langle q^\dagger i D_0 q \rangle_{\rho_N} - \frac{c_4}{3q^2} \langle q^\dagger q \rangle_{\rho_N}^2 + \cdots \; , \tag{3.74}$$

where all polynomials in q^2 and q_0^2, which vanish under the Borel transform, have been omitted, and the terms proportional to quark masses have been neglected. Note that the contributions from four-quark condensates are included in factorized form. We work in the rest frame of the nuclear matter and have defined

$$c_1 = 7t^2 - 2t - 5 \; , \qquad c_2 = 1 - t^2 \; , \qquad c_3 = 2t^2 - t - 1 \; , \tag{3.75}$$

$$c_4 = 5t^2 + 2t + 5 \; , \qquad c_5 = 7t^2 + 10t + 7 \; . \tag{3.76}$$

E. Estimating QCD Condensates

1. Vacuum Condensates

Before considering finite-density condensates, we review standard estimates of vacuum condensates. The Lorentz invariance of the vacuum state $|0\rangle$ dictates that only spin-0 operators can have nonvanishing vacuum expectation values; the lowest-dimensional vacuum condensates are

$$
\begin{aligned}
&\langle\overline{q}q\rangle_{\text{vac}}\ ,\ \langle\overline{s}s\rangle_{\text{vac}}\ , && d=3\ ,\\
&\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\text{vac}}\ , && d=4\ ,\\
&\langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_{\text{vac}}\ ,\ \langle g_s\overline{s}\sigma\cdot\mathcal{G}s\rangle_{\text{vac}}\ , && d=5\ ,\\
&\langle\overline{u}\Gamma_1 u\,\overline{u}\Gamma_2 u\rangle_{\text{vac}}\ ,\ \langle\overline{u}\Gamma_1\lambda^A u\,\overline{u}\Gamma_2\lambda^A u\rangle_{\text{vac}}\ ,\ldots\ , && d=6\ ,\\
&\langle g_s^3 fG^3\rangle_{\text{vac}}\ , && d=6\ ,
\end{aligned}
\tag{3.77}
$$

where d denotes the mass dimension of the condensate. We take $\sigma_{\mu\nu}\equiv i[\gamma_\mu,\gamma_\nu]/2$; we have used the notation $\langle\widehat{O}\rangle_{\text{vac}}\equiv\langle 0|\widehat{O}|0\rangle$, $G^2\equiv G^A_{\mu\nu}G^{A\mu\nu}$, $\sigma\cdot\mathcal{G}\equiv\sigma_{\mu\nu}\mathcal{G}^{\mu\nu}$, and $fG^3\equiv f^{ABC}G^{A\mu}_{\lambda}G^{B\nu}_{\mu}G^{C\lambda}_{\nu}$. Other condensates, such as $\langle\overline{q}D^2q\rangle_{\text{vac}}$, can be related to those listed in Eq. (3.77) by using the field equations. We will omit discussion of $\langle g_s\overline{s}\sigma\cdot\mathcal{G}s\rangle_{\text{vac}}$ and $\langle g_s^3 fG^3\rangle_{\text{vac}}$, which will not play a role in our sum rules.

We first consider the quark condensate $\langle\overline{q}q\rangle_{\text{vac}}$. Due to isospin symmetry one has

$$
\langle\overline{q}q\rangle_{\text{vac}}\simeq\langle\overline{u}u\rangle_{\text{vac}}\simeq\langle\overline{d}d\rangle_{\text{vac}}\ . \tag{3.78}
$$

The numerical value of $\langle\overline{q}q\rangle_{\text{vac}}$ can be determined from the Gell-Mann–Oakes–Renner relation,

$$
(m_u+m_d)\langle\overline{q}q\rangle_{\text{vac}}=-m_\pi^2 f_\pi^2\,[1+O(m_\pi^2)]\ , \tag{3.79}
$$

where m_π and f_π are the pion mass and pion decay constant, and m_u and m_d are the up and down current quark masses. Both sides of Eq. (3.79) are renormalization-group invariant [101]; therefore, given the current quark masses at a particular renormalization scale, one can determine the quark condensate at that same scale. We take $m_\pi=138\,\text{MeV}$ and $f_\pi=93\,\text{MeV}$; using the standard values of the light quark masses, one obtains $m_u+m_d=14\pm 4\,\text{MeV}$ at a renormalization scale of $1\,\text{GeV}$ [102]. Thus one has

$$
\langle\overline{q}q\rangle_{\text{vac}}\simeq-(0.225\pm 0.025\,\text{GeV})^3 \tag{3.80}
$$

at a renormalization scale of $1\,\text{GeV}$ [102]. The value of the strange quark condensate is usually specified in terms of the up and down quark condensate; we take

$$
\langle\overline{s}s\rangle_{\text{vac}}=f_s\langle\overline{q}q\rangle_{\text{vac}}\ , \tag{3.81}
$$

with $f_s \simeq 0.8$ [103,8,104].

The gluon condensate was first estimated from an analysis of leptonic decays of ρ^0 and ϕ^0 mesons [105] and from a sum-rule analysis of the charmonium spectrum [5]. Its numerical value is taken to be [106]

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rm vac} \simeq (0.33 \pm 0.04\,\mathrm{GeV})^4 \ . \tag{3.82}$$

(Also see Ref. [106] for a discussion of lattice QCD extractions of the gluon condensate.) Note that the product $(\alpha_s/\pi)G^2$ is approximately renormalization-group invariant; violations of renormalization-group invariance are of higher order in α_s [101].

The quark-gluon condensate $\langle g_s \overline{q} \sigma \cdot \mathcal{G} q\rangle_{\rm vac}$ is expressed in terms of the quark condensate $\langle \overline{q} q\rangle_{\rm vac}$:

$$\langle g_s \overline{q} \sigma \cdot \mathcal{G} q\rangle_{\rm vac} = 2\langle \overline{q} D^2 q\rangle_{\rm vac} \equiv 2\lambda_q^2 \langle \overline{q} q\rangle_{\rm vac} \ , \tag{3.83}$$

where we have used Eqs. (2.9) and (2.12) to obtain the first equality. Thus λ_q^2 parameterizes the average vacuum gluon field strength and the average virtuality (momentum squared) of the quarks in the QCD vacuum. The standard QCD sum-rule estimate of this quantity is $\lambda_q^2 = 0.4 \pm 0.1\,\mathrm{GeV}^2$ [103,70]. Somewhat larger values have been obtained from a lattice calculation ($\lambda_q^2 = 0.55 \pm 0.05\,\mathrm{GeV}^2$) [107] and a QCD sum-rule analysis of the pion form factor using nonlocal quark and gluon condensates ($\lambda_q^2 = 0.7 \pm 0.1\,\mathrm{GeV}^2$) [108]. A much larger value for λ_q^2 is obtained in a sum-rule analysis of the pion wave function using nonlocal condensates [109]. The value suggested by this analysis is $\lambda_q^2 \simeq 1.2\,\mathrm{GeV}^2$, which agrees with the value obtained with an instanton liquid model [110].

In QCD sum-rule applications, higher-dimensional condensates are usually approximated in terms of $\langle \overline{q} q\rangle_{\rm vac}$ and $\langle (\alpha_s/\pi) G^2\rangle_{\rm vac}$. For example, the four-quark condensates are frequently estimated in terms of $\langle \overline{q} q\rangle_{\rm vac}^2$ by using the factorization, or vacuum-saturation, approximation. This approximation corresponds to inserting a complete set of intermediate states in the middle of the four-quark matrix element, but retaining only the dominant vacuum intermediate state. An analogous approximation is commonly used in many-body physics [92]. The factorization approximation has been justified in large-N_c QCD [111]; in QCD with $N_c = 3$, it has been argued that the contribution to four-quark condensates from single-pion intermediate states (the lowest excitations of the vacuum) is small compared to that of the vacuum intermediate state [5]. Four-quark condensates in vacuum are thus estimated as

$$\langle \overline{u} \Gamma_1 u\, \overline{u} \Gamma_2 u\rangle_{\rm vac} = \tfrac{1}{16} \langle \overline{u} u\rangle_{\rm vac}^2 [\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\Gamma_2) - \tfrac{1}{3}\mathrm{Tr}(\Gamma_1 \Gamma_2)] \ , \tag{3.84}$$

$$\langle \overline{u} \Gamma_1 \lambda^A u\, \overline{u} \Gamma_2 \lambda^A u\rangle_{\rm vac} = -\tfrac{1}{9} \langle \overline{u} u\rangle_{\rm vac}^2 \mathrm{Tr}(\Gamma_1 \Gamma_2) \ , \tag{3.85}$$

and so on, where Γ_1 and Γ_2 are Dirac matrices, and Tr denotes a trace over Dirac indices. A more detailed discussion of the factorization approximation and the estimation of four-quark condensates at finite density, including those of mixed flavor (not shown here), is in Ref. [20]. Note that phenomenological studies [112,113] and instanton liquid models [106] suggest strong deviations from the factorized results in some cases.

Thus there are a small number of condensates up to dimension six that could contribute to nucleon sum rules in the vacuum. To generalize the sum rules to finite density, the density dependence of these condensates must be estimated. In addition, there are a number of new condensates that vanish in the vacuum, but are nonzero in nuclear matter.

To calculate the nucleon correlator at finite density, we need to know the condensates in nuclear matter. A detailed discussion of how these can be estimated is given in Ref. [20]. Here we focus on the dominant condensates and outline how others can be determined.

We work in the rest frame, where $u^\mu \to u'^\mu \equiv (1, \mathbf{0})$, and expand the in-medium condensates in terms of the rest-frame nucleon density. To first order in the nucleon density, we have

$$\langle \widehat{O} \rangle_{\rho_N} = \langle \widehat{O} \rangle_{\text{vac}} + \langle \widehat{O} \rangle_N \rho_N + \cdots \ , \tag{3.86}$$

where $\cdots$ denotes correction terms that are of higher order in the nucleon density. Note that this expansion is *not* a Taylor series expansion in ρ_N, since the next term in the expansion is not $O(\rho_N^2)$. The spin-averaged nucleon matrix element is

$$\langle \widehat{O} \rangle_N = \int_V d^3x \, (\langle \widetilde{N} | \widehat{O} | \widetilde{N} \rangle - \langle 0 | \widehat{O} | 0 \rangle) \ , \tag{3.87}$$

where $|\widetilde{N}\rangle$ is the state vector for a nucleon at rest normalized to unity ($\langle \widetilde{N} | \widetilde{N} \rangle = 1$) in a box of volume V. Using more conventional notation, the nucleon matrix element is given by

$$\langle \widehat{O} \rangle_N = \langle N | \widehat{O} | N \rangle \ , \tag{3.88}$$

where $|N\rangle$ is once again the state vector for a nucleon at rest. In this case, the connected matrix element is implied, which is equivalent to making a vacuum subtraction as in Eq. (3.87), and the nucleon plane-wave states are normalized as follows:

$$\langle N(p) | N(p') \rangle = \frac{\omega_p}{M_N} (2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p}') \ , \tag{3.89}$$

with $\omega_p = p_0 = \sqrt{\mathbf{p}^2 + M_N^2}$.

For a general operator $\widehat{O}$, there is not a systematic way to study contributions to $\langle \widehat{O} \rangle_{\rho_N}$ that are of higher order in ρ_N. In the case of $\langle \overline{q}q \rangle_{\rho_N}$, however, higher-order corrections can be systematically studied with an application of the Hellmann-Feynman theorem (see Sec. III C), although the corrections are necessarily model dependent. Estimates of $\langle \overline{q}q \rangle_{\rho_N}$ in Ref. [51] imply that the linear approximation is reasonably good (higher-order corrections $\sim 20\%$ of the linear term) up to nuclear matter saturation density, although this cannot be considered a definitive conclusion. Without further justification, we assume that the first-order approximation for *all* the condensates is good up to nuclear matter saturation density. We now proceed to estimate the in-medium condensates.

The most important condensates in finite-density QCD sum rules for baryons are the dimension-three quark condensates $\langle \overline{q}q \rangle_{\rho_N}$, $\langle \overline{s}s \rangle_{\rho_N}$ $\langle q^\dagger q \rangle_{\rho_N}$, and $\langle s^\dagger s \rangle_{\rho_N}$. The quark condensates alone contribute to the leading-order sum-rule results for the baryon self-energies [14,18,22,27].

The simplest dimension-three quark condensates are $\langle q^\dagger q \rangle_{\rho_N}$ and $\langle s^\dagger s \rangle_{\rho_N}$. Since the baryon current is conserved, these condensates are proportional to the net nucleon and strangeness densities, respectively:

$$\langle q^\dagger q\rangle_{\rho_N} = \tfrac{3}{2}\rho_N \ , \ \langle s^\dagger s\rangle_{\rho_N} = 0 \ . \tag{3.90}$$

These are exact results.

The in-medium quark condensate $\langle \overline{q}q\rangle_{\rho_N}$ can be expanded in terms of the nucleon density as

$$\langle \overline{q}q\rangle_{\rho_N} = \langle \overline{q}q\rangle_{\rm vac} + \langle \overline{q}q\rangle_N \rho_N + \cdots \ , \tag{3.91}$$

where we have used Eq. (3.86). This condensate has been discussed in Sec. III C and more extensively in Refs. [51,12,17]; for completeness, we simply quote the result again here. The nucleon matrix element $\langle \overline{q}q\rangle_N$ is related to the nucleon σ term $\sigma_N \equiv (m_u + m_d)\langle \overline{q}q\rangle_N$, where m_u and m_d are the up and down current quark masses. Combined with the Gell-Mann–Oakes–Renner relation [Eq (3.79)], one obtains

$$\frac{\langle \overline{q}q\rangle_{\rho_N}}{\langle \overline{q}q\rangle_{\rm vac}} = 1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} + \cdots \ . \tag{3.92}$$

The most recent estimate of the σ term is $\sigma_N \simeq 45 \pm 10\,\mathrm{MeV}$ [82]; thus the in-medium quark condensate is 30–45% smaller than its vacuum value at nuclear matter saturation density.

The strange quark condensate $\langle \overline{s}s\rangle_{\rho_N}$ is expanded in a similar manner:

$$\langle \overline{s}s\rangle_{\rho_N} = \langle \overline{s}s\rangle_{\rm vac} + \langle \overline{s}s\rangle_N \rho_N + \cdots \ . \tag{3.93}$$

The nucleon matrix element $\langle \overline{s}s\rangle_N$ is commonly parameterized by the dimensionless quantity $y \equiv \langle \overline{s}s\rangle_N / \langle \overline{q}q\rangle_N$. Calculations that analyze the mass spectrum of the baryon octet in the context of SU(3) flavor symmetry indicate that the σ term is related to the strangeness content y in the following manner [82]:

$$\sigma_N = \frac{\sigma_N^0}{1-y} \ , \tag{3.94}$$

where σ_N^0 is the σ term in the limit of vanishing strangeness content. Thus one obtains

$$\frac{\langle \overline{s}s\rangle_{\rho_N}}{\langle \overline{s}s\rangle_{\rm vac}} = 1 - \frac{(\sigma_N - \sigma_N^0)\rho_N}{f_s m_\pi^2 f_\pi^2} + \cdots \ . \tag{3.95}$$

An analysis of σ_N^0 based on second-order perturbation theory in $m_s - m_q$ yields $\sigma_N^0 \simeq 35 \pm 5\,\mathrm{MeV}$ [82]; hence, the in-medium strange quark condensate is 0–25% smaller than its vacuum value at nuclear matter saturation density.

Next we consider dimension-four quark condensates of the form $\langle q^\dagger i D_0 q\rangle_{\rho_N}$ and $\langle (\alpha_s/\pi) G^2\rangle_{\rho_N}$ [note that $\langle \overline{q} i D_0 q\rangle_{\rho_N}$ follows from Eq. (3.58) and $\langle (\alpha_s/\pi)(\mathbf{E}^2 + \mathbf{B}^2)\rangle_{\rho_N}$ is numerically unimportant]. The dimension-four condensates are expanded to first order in the nucleon density using Eq. (3.86). In order to implement this expansion, one must first determine the vacuum values of these condensates. For example, the vacuum value of the strange quark condensate $\langle s^\dagger i D_0 s\rangle_{\rho_N}$ is given by

$$\langle s^\dagger i D_0 s\rangle_{\rm vac} = u'_\mu u'_\nu \langle \overline{s}\gamma^\mu i D^\nu s\rangle_{\rm vac} = \frac{m_s}{4}\langle \overline{s}s\rangle_{\rm vac} \ , \tag{3.96}$$

where $u'_\mu \equiv (1, \mathbf{0})$. We have used the fact that $\langle \overline{s}\gamma^\mu iD^\nu s\rangle_{\rm vac}$ can only be proportional to $g^{\mu\nu}$. The vacuum values of the other condensates are determined by similar considerations. Thus the remaining dimension-four condensates are expanded as follows:

$$\langle q^\dagger iD_0 q\rangle_{\rho_N} = \langle q^\dagger iD_0 q\rangle_N \rho_N + \cdots \ , \tag{3.97}$$

$$\langle s^\dagger iD_0 s\rangle_{\rho_N} = \frac{m_s}{4}\langle \overline{s}s\rangle_{\rm vac} + \langle s^\dagger iD_0 s\rangle_N \rho_N + \cdots \ , \tag{3.98}$$

$$\left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle_{\rho_N} = \left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle_{\rm vac} + \left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle_N \rho_N + \cdots \ . \tag{3.99}$$

The QCD trace anomaly is used to estimate $\langle(\alpha_s/\pi)G^2\rangle_N$. The details are discussed in Refs. [51,12,17]; therefore, we simply quote the result here:

$$\left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle_N = -\tfrac{8}{9}(M_N - \sigma_N - S_N) \ , \tag{3.100}$$

where M_N is the nucleon mass, $\sigma_N \equiv (m_u + m_d)\langle \overline{q}q\rangle_N$ is the nucleon σ term, and we define $S_N \equiv m_s\langle \overline{s}s\rangle_N$. From Eq. (3.94), S_N can be parameterized as

$$S_N = \left(\frac{m_s}{m_u + m_d}\right)(\sigma_N - \sigma_N^0) \ . \tag{3.101}$$

We take $m_s/(m_u+m_d) \simeq 13$ [82]; thus we have the following estimate for the nucleon matrix element in Eq. (3.100):

$$\left\langle \frac{\alpha_s}{\pi}G^2 \right\rangle_N \simeq -0.650 \pm 0.150\,\text{GeV} \ . \tag{3.102}$$

At nuclear matter saturation density, $\langle(\alpha_s/\pi)G^2\rangle_{\rho_N}$ is about 5–10% smaller than its vacuum value .

The matrix elements $\langle q^\dagger iD_0 q\rangle_N$ and $\langle s^\dagger iD_0 s\rangle_N$ can be related to moments of parton distribution functions measured in deep-inelastic scattering experiments [19,12,17]:

$$\langle q^\dagger iD_0 q\rangle_N = \tfrac{3}{8}M_N A_2^q(\mu^2) \simeq 0.18 \pm 0.01\,\text{GeV} \ , \tag{3.103}$$

$$\langle s^\dagger iD_0 s\rangle_N = \tfrac{1}{4}S_N + \tfrac{3}{8}M_N A_2^s(\mu^2) \simeq 0.06 \pm 0.04\,\text{GeV} \ , \tag{3.104}$$

where the renormalization scale $\mu = 1\,\text{GeV}$ is used. The moments of the parton distribution functions are defined as [114–116]

$$A_n^q(\mu^2) = 2\int_0^1 dx\, x^{n-1}[q(x,\mu^2) + (-1)^n \overline{q}(x,\mu^2)] \ , \tag{3.105}$$

$$A_n^s(\mu^2) = 2\int_0^1 dx\, x^{n-1}[s(x,\mu^2) + (-1)^n \overline{s}(x,\mu^2)] \ , \tag{3.106}$$

where $q(x,\mu^2)$, $s(x,\mu^2)$, $\overline{q}(x,\mu^2)$, and $\overline{s}(x,\mu^2)$ are the scale-dependent distribution functions for quarks and antiquarks in the nucleon [117,118]. See Ref. [20] for more details of these estimates.

We can evaluate dimension-five condensates through a combination of parton distributions and model calculations. With the exception of

$$\langle q^\dagger iD_0\, iD_0 q\rangle_N + \tfrac{1}{12}\langle g_s q^\dagger \sigma\cdot\mathcal{G} q\rangle_N \simeq 0.031\,\mathrm{GeV}^2\ , \tag{3.107}$$

which is obtained from parton distribution functions, none of the dimension-five condensates have been determined accurately; however, terms proportional to these condensates make only small contributions to the nucleon sum rules. Thus the sensitivity of our sum-rule results to the precise values of these condensates is small. A numerical analysis of this sensitivity is given in Ref. [25].

F. Results and Qualitative Conclusions

In this section, we analyze the sum rules for nucleons in infinite nuclear matter with a general interpolating field. In the operator product expansion (OPE) for the nucleon correlator, we work to leading order in perturbation theory; leading-logarithmic corrections are included through anomalous-dimension factors. Contributions proportional to the up and down current quark masses are neglected as they give numerically small contributions. In the numerical results, we include pure gluon condensates up to dimension four and quark and gluon condensates up to dimension five. At dimension six, we include only the four quark condensates, which give numerically important contributions to nucleon sum rule in vacuum and in nuclear matter. All other dimension-six and higher dimensional condensates are neglected since their contributions are expected to be small.

1. Borel Sum Rules

QCD sum rules for the nucleon follow by equating the phenomenological representation to the OPE representation. More generally, we can exploit the analytic structure of the correlator by considering integrals over contours running above and below the real axis, and then closing in the upper and lower half planes, respectively (see Ref. [18]). By approximating the correlator in the different regions of integration and applying Cauchy's theorem, we can derive a general class of sum rules, which manifest the duality between the physical hadronic spectrum and the spectral function calculated in a QCD expansion:

$$\int_{-\overline{\omega}_0}^{\omega_0} d\omega\, W(\omega)\rho^{\mathrm{phen}}(\omega,|\mathbf{q}|) - \int_{-\overline{\omega}_0}^{\omega_0} d\omega\, W(\omega)\rho^{\mathrm{OPE}}(\omega,|\mathbf{q}|) = 0\ . \tag{3.108}$$

Here $W(\omega)$ is a smooth (entire) weighting function and the spectral densities ρ^{phen} and ρ^{OPE} are proportional to the discontinuities of the invariant functions across the real axis. (These sum rules can also be derived by expanding dispersion relations for retarded and advanced correlators with external frequency ω' in the limit $\omega' \to i\infty$.) The phenomenological spectral density ρ^{phen} models the low-energy physical spectrum, while the theoretical spectral density ρ^{OPE} follows from the the operator product expansion (OPE). The QCD sum-rule approach assumes that, with suitable choices for W and the effective continuum thresholds ω_0 and $-\overline{\omega}_0$, each integral can be reliably calculated and meaningful results extracted.

In principle, the effective thresholds are different for positive (ω_0) and negative ($\overline{\omega}_0$) energies and for the different sum rules. The former differences are critical in some sum-rule formulations [80], but are not numerically important in our formulation. Furthermore, the

thresholds are relatively poorly determined by the sum rules and effects due to different thresholds in different sum rules may be absorbed by slight changes in the other parameters. In the present discussion, we use a universal effective threshold ω_0 for simplicity.

If we choose the weighting function $W(\omega) = \omega e^{-\omega^2/M^2}$, then the vacuum Borel sum rule is reproduced in the zero-density limit up to an overall factor of $e^{-\mathbf{q}^2/M^2}$. This is not an optimal choice at finite density, because it weights positive and negative ω equally. In order to suppress the negative-energy contribution, we use the weighting function

$$W(\omega) = (\omega - \overline{E}_q)\, e^{-\omega^2/M^2} \ , \tag{3.109}$$

where $\overline{E}_q$ is the energy of the negative-energy pole in our quasiparticle ansatz [see Eq. (3.11)]. This choice suppresses a sharp excitation completely but also strongly suppresses (relative to the positive-energy contribution) a broad excitation in this vicinity. Furthermore, this choice reduces to the usual Borel sum rule in the vacuum: The vacuum spectral densities are odd in ω; thus the $\overline{E}_q$ contribution vanishes.

The spectral densities in Eq. (3.108) can be extracted from the discontinuities in Eqs. (3.39)–(3.41) and (3.69)–(3.74). Alternatively, sum rules for the nucleon with the weighting function in Eq. (3.109) can be constructed as follows:

$$\mathcal{B}[\Pi_i^{\mathrm{E}}(q_0^2, |\mathbf{q}|) - \overline{E}_q \Pi_i^{\mathrm{O}}(q_0^2, |\mathbf{q}|)]_{\mathrm{OPE}} = \mathcal{B}[\Pi_i^{\mathrm{E}}(q_0^2, |\mathbf{q}|) - \overline{E}_q \Pi_i^{\mathrm{O}}(q_0^2, |\mathbf{q}|)]_{\mathrm{phen}} \ , \tag{3.110}$$

for $i = \{s, q, u\}$, where the left-hand side is obtained from the OPE, the right-hand side from the phenomenological dispersion relations, and $\mathcal{B}$ is the Borel transform operator defined by

$$\mathcal{B}[f(q_0^2, |\mathbf{q}|)] \equiv \lim_{\substack{-q_0^2, n \to \infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q_0^2}\right)^n f(q_0^2, |\mathbf{q}|) \equiv \widehat{f}(M^2, |\mathbf{q}|) \ . \tag{3.111}$$

The only difference from a Borel transform with respect to $Q^2 = -q^2$ is a factor of $e^{-\mathbf{q}^2/M^2}$ common to all terms, which will cancel. In the zero-density limit, contributions from the second term in Eq. (3.109) vanish, and we once again recover the usual vacuum sum rules.

Perturbative corrections $\sim \alpha_s^n$ can be taken into account in the leading logarithmic approximation through anomalous-dimension factors [5]. After the Borel transform, the effect of these corrections is to multiply each term on the OPE side by the factor [5,6,18]

$$L^{-2\Gamma_\eta + \Gamma_{O_n}} \equiv \left[\frac{\ln(M/\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\mathrm{QCD}})}\right]^{-2\Gamma_\eta + \Gamma_{O_n}} \ , \tag{3.112}$$

where Γ_η is the anomalous dimension of the interpolating field η, Γ_{O_n} is the anomalous dimension of the corresponding local operator (including the current quark masses), μ is the normalization point of the operator product expansion, and Λ_{QCD} is the QCD scale parameter. In general, the absolute predictions of QCD sum rules are sensitive to the precise choices of μ and Λ_{QCD}. However, when ratios of finite-density to zero-density quantities are taken, as we do here, the predictions are insensitive for $100\,\mathrm{MeV} < \Lambda_{\mathrm{QCD}} < 200\,\mathrm{MeV}$ [25].[16]

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[16] Values of Λ_{QCD} much larger than 200 MeV, as obtained from some experimental analyses, are problematic for the QCD sum rules (at any density). Shifman has recently argued that such values are incompatible with crucial features of QCD and should be regarded with skepticism [119].

Applying Eq. (3.110) to the ansätze in Eqs. (3.39)–(3.41) and the OPE results in Eqs. (3.69)–(3.74), we obtain three sum rules—one for each invariant function:

$$\lambda_N^{*2} M_N^* e^{-(E_q^2-\mathbf{q}^2)/M^2} = -\frac{c_1}{16\pi^2} M^4 E_1 \langle\overline{q}q\rangle_{\rho_N} - \frac{c_1}{3}\overline{E}_q \langle\overline{q}q\rangle_{\rho_N} \langle q^\dagger q\rangle_{\rho_N} + \cdots \ , \tag{3.113}$$

$$\begin{aligned}
\lambda_N^{*2} e^{-(E_q^2-\mathbf{q}^2)/M^2} &= \frac{c_4}{256\pi^4} M^6 E_2 L^{-4/9} + \frac{c_4}{24\pi^2}\overline{E}_q M^2 E_0 \langle q^\dagger q\rangle_{\rho_N} L^{-4/9} \\
&\quad - \frac{c_4}{72\pi^2} M^2 \left(5E_0 - \frac{8\mathbf{q}^2}{M^2}\right) \langle q^\dagger iD_0 q\rangle_{\rho_N} L^{-4/9} \\
&\quad + \frac{c_4}{256\pi^2} M^2 E_0 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho_N} L^{-4/9} \\
&\quad + \frac{c_1}{6} \langle\overline{q}q\rangle_{\rho_N}^2 L^{4/9} + \frac{c_4}{6} \langle q^\dagger q\rangle_{\rho_N}^2 L^{-4/9} + \cdots \ ,
\end{aligned} \tag{3.114}$$

$$\begin{aligned}
\lambda_N^{*2} \Sigma_v e^{-(E_q^2-\mathbf{q}^2)/M^2} &= \frac{c_4}{12\pi^2} M^4 E_1 \langle q^\dagger q\rangle_{\rho_N} L^{-4/9} \\
&\quad + \frac{5c_4}{18\pi^2}\overline{E}_q M^2 E_0 \langle q^\dagger iD_0 q\rangle_{\rho_N} L^{-4/9} + \frac{c_4}{3}\overline{E}_q \langle q^\dagger q\rangle_{\rho_N}^2 L^{-4/9} + \cdots \ .
\end{aligned} \tag{3.115}$$

Here we have defined

$$E_0 \equiv 1 - e^{-s_0^*/M^2} \ , \tag{3.116}$$

$$E_1 \equiv 1 - e^{-s_0^*/M^2}\left(\frac{s_0^*}{M^2} + 1\right) \ , \tag{3.117}$$

$$E_2 \equiv 1 - e^{-s_0^*/M^2}\left(\frac{s_0^{*2}}{2M^4} + \frac{s_0^*}{M^2} + 1\right) \ , \tag{3.118}$$

where we define $s_0^* \equiv \omega_0^2 - \mathbf{q}^2$. These factors arise from the continuum model, which approximates the contributions from higher-energy states by the OPE spectral density, starting at a sharp energy threshold ω_0.

We see that the above sum rules explicitly involve the Borel mass M^2. If both the QCD expansion and phenomenological description were exact, predictions for the spectral parameters would be independent of M^2. In practice, both sides are represented imperfectly. The hope is that there exists a range in M^2 for which the two sides have a good overlap.

2. Simplified Sum Rules

The Ioffe formula [Eq. (2.58)] manifests the principal physical content of the nucleon sum rule in vacuum in a highly truncated form, which is justified *a posteriori* by examining the full sum rule. Here we construct the analogous simplified finite-density sum rules for the nucleon, which follow by keeping in each of the three sum rules of Eqs. (3.113)–(3.115) only the quasinucleon-pole contribution to the phenomenological side (*i.e.*, no continuum

factors) and only the leading term in the operator product expansion on the OPE side (without anomalous-dimension corrections):

$$\lambda_N^2 M_N^* e^{-(E_q^2-\mathbf{q}^2)/M^2} = -\frac{1}{4\pi^2} M^4 \langle \overline{q} q \rangle_{\rho_N} \ , \tag{3.119}$$

$$\lambda_N^2 e^{-(E_q^2-\mathbf{q}^2)/M^2} = \frac{1}{32\pi^4} M^6 \ , \tag{3.120}$$

$$\lambda_N^2 \Sigma_v e^{-(E_q^2-\mathbf{q}^2)/M^2} = \frac{2}{3\pi^2} M^4 \langle q^\dagger q \rangle_{\rho_N} \ . \tag{3.121}$$

Here Ioffe's current [Eq. (2.14)], *i.e.*, Eq. (3.26) with $t = -1$, has been used. By considering ratios of these sum rules evaluated at a value of the Borel mass in the middle of the range where the full sum rules will be considered, one hopes to extract the basic physics. As one can see later, this truncation is qualitatively reasonable *except* for the large contribution of four-quark condensates.

Taking ratios of Eqs. (3.119)–(3.121), one obtains the simple expressions

$$M_N^* = -\frac{8\pi^2}{M^2} \langle \overline{q} q \rangle_{\rho_N} \ , \tag{3.122}$$

$$\Sigma_v = \frac{64\pi^2}{3M^2} \langle q^\dagger q \rangle_{\rho_N} \ , \tag{3.123}$$

which might be expected to apply up to nuclear matter saturation density. Using Eqs. (3.90) and (3.92), one can determine the scalar and vector self-energies in terms of the nucleon density:

$$\Sigma_s = -\frac{4\pi^2}{M^2} \frac{\sigma_N \rho_N}{m_q} \ , \tag{3.124}$$

$$\Sigma_v = \frac{32\pi^2}{M^2} \rho_N \ . \tag{3.125}$$

One observes that both Σ_s and Σ_v are proportional to ρ_N. Taking the ratio of Eqs. (3.124) and (3.125), one finds that the explicit dependence of the self-energies on the Borel mass and the density drops out:

$$\frac{\Sigma_s}{\Sigma_v} = -\frac{\sigma_N}{8 m_q} \ . \tag{3.126}$$

For typical values of σ_N and light quark masses, this ratio is close to -1, indicating a substantial cancellation of Σ_s and Σ_v in the medium. Thus the predictions of the simplest sum rules are in qualitative agreement with several features of relativistic phenomenology: the self-energies scale with the density, they are weakly dependent on the nucleon state (three-momentum), and scalar and vector self-energies cancel.

Alternatively, one can normalize the self-energies to the nucleon mass determined by taking the zero-density limit of Eq. (3.122):

$$M_N = -\frac{8\pi^2}{M^2} \langle \overline{q} q \rangle_{\text{vac}} \ . \tag{3.127}$$

The hope is that this will reduce the sensitivity to particular details of the sum rules and to the level of truncation, provided one works to the same level of approximation at finite and zero density. Adopting the same Borel mass for both finite and zero density and taking ratios, one obtains results independent of M^2:

$$\frac{M_N^*}{M_N} = 1 + \frac{\Sigma_s}{M_N} = \frac{\langle\overline{q}q\rangle_{\rho_N}}{\langle\overline{q}q\rangle_{\rm vac}} = 1 - \frac{\sigma_N \rho_N}{m_\pi^2 f_\pi^2} \ , \tag{3.128}$$

$$\frac{\Sigma_v}{M_N} = -\frac{8}{3}\frac{\langle q^\dagger q\rangle_{\rho_N}}{\langle\overline{q}q\rangle_{\rm vac}} = \frac{8 m_q \rho_N}{m_\pi^2 f_\pi^2} \ . \tag{3.129}$$

The last equalities in Eqs. (3.128) and (3.129) follow from Eqs. (3.90), (3.92), and (3.79). [Note: The independence of M^2 in the ratios in Eqs. (3.128) and (3.129) should not be interpreted as evidence that the *individual* sum rules are weakly dependent on M^2.] For typical values of the relevant condensates and other parameters, $M_N^*/M_N \sim 0.6$–0.7 and $\Sigma_v/M_N \sim 0.3$–0.4. This is in good agreement with the values used in relativistic mean-field models that provide good fits to bulk properties of finite nuclei [39].

The key feature that assures qualitative agreement with relativistic phenomenology is that Eq. (3.120) is density independent to leading order. In the simple sum rule, this implies that the pole position and residue do not vary much with density. This in turn implies the results of Eqs. (3.122) and (3.123), in which the effective mass naturally follows $\langle\overline{q}q\rangle_{\rho_N}$ and the vector self-energy follows $\langle q^\dagger q\rangle_{\rho_N}$. In the more complete sum-rule analysis considered below, these basic results survive if the correction terms to Eqs. (3.119) and (3.121) are not overly large *and* if Eq. (3.120) remains weakly density dependent. The latter condition turns out to be problematic.

3. Detailed Sum-Rule Analysis

In principle, the predictions based on sum rules should be independent of the auxiliary parameter M^2. In practice, however, one has to truncate the OPE and use a simple phenomenological ansatz for the spectral density, so one expects at best that the two descriptions overlap only in a limited range of M^2. As a result, one expects to see a "plateau" in the predicted quantities as functions of M^2 (although not necessarily a local extremum). Nucleon sum rules in vacuum truncated at dimension-six condensates do not provide a plateau [18,68,104]; nevertheless, it will be assumed here that the sum rules have a region of overlap, although imperfect.[17] We normalize the finite-density predictions for all self-energies to the zero-density prediction for the mass, with the expectation that this will compensate for at least some of the limitations of the truncated sum rules.

To analyze the sum rules and extract the self-energies, one can sample the sum rules in the fiducial region, which is the overlap between the region where the sum rule is dominated by the quasinucleon contribution and the region where the truncated OPE is reliable. In

[17]Including direct-instanton effects in nucleon sum rules in vacuum leads to a more convincing plateau [72,73].

choosing the fiducial region, one may introduce a lower bound of the Borel mass such that the highest-dimensional condensate contributes no more than $\sim$ 10% to the total of the terms on the right-hand sides of Eqs. (3.113)–(3.115) and an upper bound such that the continuum contribution is less than $\sim$ 50% of the total phenomenological contribution (*i.e.*, the sum of the quasinucleon pole contribution and the continuum contribution). To quantify the fit of the left- and right-hand sides, one can apply the logarithmic measure

$$\delta(M^2) = \ln\left[\frac{\max\{\lambda_N^{*2} e^{-(E_q^2-\mathbf{q}^2)/M^2}, \Pi'_s/M_N^*, \Pi'_q, \Pi'_u/\Sigma_v\}}{\min\{\lambda_N^{*2} e^{-(E_q^2-\mathbf{q}^2)/M^2}, \Pi'_s/M_N^*, \Pi'_q, \Pi'_u/\Sigma_v\}}\right] , \tag{3.130}$$

which we average over 150 points evenly spaced within the fiducial region of M^2. Here Π'_s, Π'_q, and Π'_u denote the right-hand sides of Eqs. (3.113)–(3.115), respectively. The predictions for M_N^*, Σ_v, s_0^*, and λ_N^{*2} are obtained by minimizing the averaged measure δ. This approach weights the fits in the region where the continuum contribution is minimal and reduces the sensitivity to the endpoints of the optimum region [104]. To get a prediction for the nucleon mass in vacuum, one applies the same procedure to the sum rules evaluated in the zero-density limit.

In the analysis to follow, the quasinucleon three-momentum is fixed at $|\mathbf{q}| = 270\,\mathrm{MeV}$ (*i.e.*, approximately the Fermi momentum) and the nucleon σ term is taken to be $\sigma_N = 45\,\mathrm{MeV}$. The dimension-five nucleon matrix elements not given earlier are taken to be [25]

$$\langle \overline{q} iD_0\, iD_0 q\rangle_N + \tfrac{1}{8}\langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_N = 0.3\,\mathrm{GeV}^2 , \tag{3.131}$$

$$\langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_N = 3.0\,\mathrm{GeV}^2 , \tag{3.132}$$

$$\langle g_s q^\dagger\sigma\cdot\mathcal{G}q\rangle_N = -0.33\,\mathrm{GeV}^2 . \tag{3.133}$$

The values of vacuum condensates are taken to be $\langle\overline{q}q\rangle_{\mathrm{vac}} = -(245\,\mathrm{MeV})^3$, $\langle(\alpha_s/\pi)G^2\rangle_{\mathrm{vac}} = (330\,\mathrm{MeV})^4$, and $\langle g_s\overline{q}\sigma\cdot\mathcal{G}q\rangle_{\mathrm{vac}} = m_0^2\langle\overline{q}q\rangle_{\mathrm{vac}}$ with $m_0^2 = 0.8\,\mathrm{GeV}^2$. Nuclear matter saturation density is taken to be $\rho_N = (110\,\mathrm{MeV})^3$.

Four-quark condensates are numerically important in both the vacuum and the finite-density nucleon sum rules, because they contribute in tree diagrams and do not carry the numerical suppression factors typically associated with loops. In the sum rules in Eqs. (3.113)–(3.115), the contributions from the four-quark condensates in their in-medium factorized forms are included; however, the factorization approximation may not be justified in nuclear matter. In the case of the "scalar-vector" and "vector-vector" four-quark condensates, $\langle\overline{q}q\rangle_{\rho_N}\langle q^\dagger q\rangle_{\rho_N}$ and $\langle q^\dagger q\rangle_{\rho_N}^2$, such concerns are unimportant, since these condensates give minimal contributions to the nucleon sum rules [25]. Thus their factorized forms will be used here for simplicity. However, the "scalar-scalar" four-quark condensate $\langle\overline{q}q\rangle_{\rho_N}^2$ *does* give important contributions to the nucleon sum rules.

In its factorized form, the scalar-scalar four-quark condensate has a very strong density dependence, which may not be justified. Therefore we parameterize the density dependence in terms of a new parameter f. Specifically, the scalar-scalar four-quark condensate is parameterized so that it interpolates between its factorized form in free space and its factorized form in nuclear matter:

$$\langle\overline{q}q\rangle_{\rho_N}^2 \longrightarrow \langle\widetilde{\overline{q}q}\rangle_{\rho_N}^2 \equiv (1-f)\langle\overline{q}q\rangle_{\mathrm{vac}}^2 + f\langle\overline{q}q\rangle_{\rho_N}^2 . \tag{3.134}$$

The density dependence of the scalar-scalar four-quark condensate is then parameterized by f and the density dependence of $\langle\overline{q}q\rangle_{\rho_N}$ [see Eq. (3.92)]. The factorized condensate $\langle\overline{q}q\rangle^2_{\rho_N}$ appearing in Eq. (3.114) will be replaced by $\langle\widetilde{\overline{q}q}\rangle^2_{\rho_N}$ in the calculations to follow. Values of f in the range $0 \leq f \leq 1$ will be considered; $f = 0$ corresponds to the assumption of no density dependence, and $f = 1$ corresponds to the in-medium factorization assumption.

FIG. 2. Optimized sum-rule predictions for M_N^*/M_N, Σ_v/M_N, and E_q/M_N as functions of f, with Ioffe's interpolating field ($t = -1$).

The sum rules are analyzed with the Borel window fixed at $0.8 \leq M^2 \leq 1.4\,\mathrm{GeV}^2$, which is identified by Ioffe and Smilga [68] as the fiducial region for the nucleon sum rules in vacuum (with the contributions from up to dimension-nine condensates included). Here these boundaries are adopted as the maximal limits of applicability of the sum rules at finite density. We start from Ioffe's interpolating field (*i.e.*, $t = -1$). The optimized results for the ratios M_N^*/M_N, Σ_v/M_N, and E_q/M_N as functions of f are plotted in Fig. 2. One can see from Fig. 2 that M_N^*/M_N and E_q/M_N vary rapidly with f, while Σ_v/M_N is relatively insensitive to f. Therefore, the sum-rule prediction for the scalar self-energy depends *strongly* on the density dependence of the scalar-scalar four-quark condensate. For small values of f ($0 \leq f \leq 0.3$), the predictions are

$$M_N^*/M_N \simeq 0.63\text{–}0.72\ , \tag{3.135}$$

$$\Sigma_v/M_N \simeq 0.24\text{–}0.30\ , \tag{3.136}$$

which are comparable to typical values from relativistic phenomenology. On the other hand, for large f ($0.7 \leq f \leq 1$), one finds $\Sigma_v/M_N \simeq 0.34$–0.37, which is still reasonable. In contrast, the predictions for M_N^* and E_q turn out to be $M_N^*/M_N \simeq 0.84$–0.94 and $E_q/M_N \simeq 1.24$–1.36, which imply $\Sigma_s/M_N \simeq -(0.06$–$0.16)$ and a significant shift of the

quasinucleon pole relative to the nucleon pole in vacuum (the net self-energy is repulsive). Thus a significant density dependence of the scalar-scalar four-quark condensate leads to an essentially vanishing scalar self-energy and a strong vector self-energy with a magnitude of a few hundred MeV. The predictions for the ratios $\lambda_N^{*2}/\lambda_N^2$ and s_0^*/s_0 also depend on f. For small f, the continuum threshold s_0^* is close to the vacuum value while the residue λ_N^{*2} drops about 20% relative to the corresponding vacuum value. (Note that these quantities are relatively poorly determined by the sum rules.) For large f, the continuum threshold increases by 20–25% relative to the vacuum value and the residue increases by about 20%, implying a significant rearrangement of the spectrum. For intermediate values of f, both the continuum threshold and the residue are very close to the corresponding vacuum values.

FIG. 3. Ratios M_N^*/M_N and Σ_v/M_N as functions of Borel M^2, with optimized predictions for E_q, $\overline{E}_q$ and the continuum thresholds. The solid, dashed, and dotted curves corresponds to $f = 0$, 0.5, and 1, respectively.

From the sum rules in Eqs. (3.113)–(3.115), it is easy to see that the ratios Π_s'/Π_q' and Π_u'/Π_q' give M_N^* and Σ_v as functions of Borel M^2, and Π_s'/Π_q' in the zero-density limit yields M_N as a function of M^2. In Fig. 3, the ratios M_N^*/M_N and Σ_v/M_N are plotted as functions of M^2 for three different values of f, with E_q, $\overline{E}_q$, and the continuum threshold fixed at their optimized values. The curves for $f = 0$ and $f = 0.5$ (solid and dashed curves respectively) are quite flat in the optimum region, and thus imply a weak dependence of the predicted ratios on M^2 (even though the individual sum-rule predictions before taking ratios are not flat). For $f = 1$ (dotted curves), the ratio Σ_v/M_N is flat, indicating again a weak dependence on M^2; in contrast, M_N^*/M_N changes significantly in the region of interest.

FIG. 4. (a) The left- and right-hand sides of the finite-density sum rules as functions of Borel M^2 for $t = -1$ and $f = 0$, with the optimized values for M_N^*, Σ_v, s_0^*, and λ_N^{*2}. The four curves correspond to Π_s'/M_N^* (solid), Π_q' (dashed), Π_u'/Σ_v (dot-dashed), and $\lambda_N^{*2} e^{-(E_q^2-\mathbf{q}^2)/M^2}$ (dotted). (b) The left- and right-hand sides of the corresponding vacuum sum rules, with the optimized values for M_N, s_0, and λ_N^2. The three curves correspond to Π_s'/M_N (solid), Π_q' (dashed), and $\lambda_N^2 e^{-M_N^2/M^2}$ (dot-dashed) at the zero-density limit.

In Fig. 4(a), $\lambda_N^{*2} e^{-(E_q^2-\mathbf{q}^2)/M^2}$, Π_s'/M_N^*, Π_q', and Π_u'/Σ_v are plotted as functions of M^2

for $f = 0$, with the predicted values for M_N^*, Σ_v, s_0^*, and λ_N^{*2}. If the sum rules work well, one should expect the four curves to coincide with each other. It is found that their M^2 dependence in the region of interest turns out to be equal up to 15%. The overlap of the corresponding vacuum sum rules (*i.e.*, the zero-density limit) is illustrated in Fig. 4(b). One can see that the quality of the overlap for the finite-density sum rules is similar to that of the corresponding sum rules in vacuum. As f increases, the overlap of the sum rules gets better; however, this does not necessarily imply that the results with large f are more trustworthy, because other corrections (such as the contributions from higher-dimensional condensates, the corrections from the higher-order density dependence of condensates, *etc.*) will change the behavior of the overlap.

FIG. 5. Optimized sum-rule predictions for M_N^*/M_N and Σ_v/M_N as functions of t. The three curves correspond to $f = 0.2$ (solid), 0.5 (dashed), and 1.0 (dotted).

All of the results above use Ioffe's interpolating field ($t = -1$); now the results for the general interpolating field [Eq. (3.26)] are presented. In Fig. 5, the predicted ratios M_N^*/M_N and Σ_v/M_N as functions of t for three different values of f have been displayed. The ratio Σ_v/M_N increases as t increases; the rate of increase is essentially the same for all values of f. For $f = 1$, the ratio M_N^*/M_N decreases slowly as t increases; for $f = 0.5$, M_N^*/M_N is nearly independent of t; for $f = 0.2$, M_N^*/M_N increases slowly as t increases over the range of t that is of interest. It is found that the continuum contributions increase and the residue decreases as t increases. On the other hand, the overlap of the sum rules gets better as t increases. The predictions for the continuum thresholds depend only weakly on t. It is also found that for $f < 0.2$ and $-1.15 \leq t \leq -1.05$, the numerical optimizing procedure converges slowly and the predicted continuum threshold and residue are much smaller than those for $f \geq 0.2$. In this case, the continuum contributions dominate the sum rules, making the predictions for M_N^* and Σ_v unreliable.

The sensitivity of the sum-rule results to other factors, such as variations in the fiducial interval and changes in various condensates and parameters, have been examined in Refs. [25,100]. In general, one finds that the sum-rule predictions are largely insensitive to details, with the important exception of the assumed density dependence of the scalar-scalar four-quark condensate.

4. Qualitative Conclusions

The most concrete conclusion we can draw is that QCD sum rules predict a positive vector self-energy with a magnitude of a few hundred MeV for a quasinucleon in nuclear matter. This qualitative feature is *robust* and, for the most part, independent of the details of the calculation. For Ioffe's interpolating field and typical values of the relevant condensates and other input parameters, one obtains $\Sigma_v/M_N \simeq 0.24$–0.37, which is a range very similar to that found for vector self-energies in relativistic nuclear physics phenomenology. On the other hand, the prediction for the scalar self-energy depends strongly on the value of the in-medium scalar-scalar four-quark condensate, which is not well established, and on the value of the nucleon σ term. This means that the conclusions about the quasinucleon scalar self-energy must still be somewhat indefinite. Nevertheless, it should be emphasized that predictions with different values of the four-quark condensate give different physical features that are not equally compatible with known nuclear phenomenology.

FIG. 6. Optimized sum-rule predictions for M_N^*/M_N (diamonds) and Σ_v/M_N (squares) as functions of density, with $f = 0$ (parameters are specified in Ref. [18]). The predictions from a nonlinear relativistic mean-field model [40] are shown for comparison (dashed and dot-dashed curves).

If the scalar-scalar four-quark condensate depends only weakly on the nucleon density (*i.e.*, if f is small), the prediction for M_N^*/M_N is insensitive to the Borel mass. The predicted scalar self-energy is large and negative, which is consistent with relativistic phenomenology. In this case, there is a significant degree of cancellation between the scalar and vector self-energies, which leads to a quasinucleon energy close to the free-space nucleon mass. This result is compatible with the empirical observation that the quasinucleon energy is shifted only slightly in nuclear matter relative to the free-space mass. The density dependence of the self-energies for this case is shown in Fig. 6. The prediction for the continuum threshold is close to the vacuum value and the residue at the quasinucleon pole drops slightly relative to the corresponding vacuum value. This is also compatible with experiment; there is no evidence for a strong rearrangement of the spectrum at nuclear matter saturation density, merely a spreading of strength over energy scales too small to be resolved by the sum rules.

In contrast, if the four-quark condensate has a strong density dependence (*i.e.*, if f is large), the predicted ratio M_N^*/M_N varies strongly with the Borel mass, and the magnitude of M_N^*/M_N is close to unity, implying that the scalar self-energy is essentially zero. The predicted vector self-energy, on the other hand, is larger than it is with small f. Thus the resulting quasinucleon energy is significantly larger than the nucleon mass. This result is unrealistic and is totally different from known nuclear phenomenology. Moreover, both the continuum threshold and the residue at the quasinucleon pole are well above their values in vacuum; this suggests a significant rearrangement of the spectrum in nuclear matter, which is inconsistent with experiment.

For intermediate values of f, the predicted scalar self-energy is negative with a sizable magnitude. The vector self-energy is still strong. The magnitudes of the self-energies and the degree of cancellation between them depend on the choice of interpolating field and the values of the condensates and input parameters. The quasinucleon energy, the residue at the quasinucleon pole, and the continuum threshold are close to their corresponding vacuum values.

The qualitative features discussed above can also be identified through the dominant behavior of the OPE sides of the sum rules. From Eq. (3.114), one notes that Π_q' is mainly determined by the density-independent leading-order perturbative term and the scalar-scalar four-quark condensate. For small f, Π_q' is close to its zero-density value; this implies that the quasinucleon energy, residue, and continuum are essentially unchanged from their values in vacuum. Since Π_s' is dominated by the leading-order term proportional to the in-medium quark condensate, the significant reduction of $\langle\overline{q}q\rangle_{\rho_N}$ from its vacuum value $\langle\overline{q}q\rangle_{\rm vac}$ implies a significant reduction of M_N^* from M_N. The vector self-energy tends to follow the nucleon density, as the leading-order term proportional to $\langle q^\dagger q\rangle_{\rho_N}$ gives the largest contribution to Π_u'. For large values of f, Π_q' is significantly reduced from its vacuum value; this leads to a shift in the quasinucleon energy and a significant rearrangement of the spectrum. As Π_s' and Π_u' are independent on f, one expects M_N^* and Σ_v to increase due to the reduction of Π_q'. Clearly, further study of the in-medium four-quark condensates is very important.

The sum-rule predictions are fairly sensitive to the choice of interpolating field, reflecting the dependence of the truncated OPE on this choice. In the region $-1.15 \leq t \leq -1$, the scalar and vector self-energies (recall $\Sigma_s = M_N^* - M_N$) each have a magnitude of a few hundred MeV with opposite signs for $f < 0.5$, which is in qualitative agreement with relativistic phenomenology. As t gets larger, smaller f values are needed to produce large and canceling scalar and vector self-energies. It is worth emphasizing that, in the interval of t considered here, the contributions of higher-order terms in the OPE become more important for larger magnitudes of t, while the continuum contributions become larger and the coupling of the interpolating field to the quasinucleon states becomes weaker for smaller magnitudes of t. Since there is less information about the higher-dimensional condensates, and only the quasinucleon state is of interest here, one should not use a t with a magnitude that is too large or too small. Within the range of t considered in this section, the vector self-energy is always large; the scalar self-energy is mainly controlled by the value of f.

The nucleon σ term σ_N is a crucial phenomenological input in the finite-density nucleon sum rules; its value determines the degree of chiral restoration in the nuclear medium (to first order in the nucleon density). The scalar self-energy strongly depends on σ_N through $\langle \overline{q}q \rangle_{\rho_N}$ and the four-quark condensates [25,100]. One observes that the large and canceling self-energies found with small and moderate f values mainly depend on the ratio σ_N / m_q. An understanding of the cancellation between scalar and vector components will be essential in making connections between QCD and nuclear physics.

Even if one assumes that the scalar-scalar four-quark condensate has weak or moderate density dependence (so that the sum-rule predictions are consistent with known relativistic phenomenology), there are still important open questions to confront. One must test sum rules for other baryons as well as for other nucleon properties. In Sec. IV, the finite-density sum-rule approach will be extended to study the self-energies of hyperons in nuclear matter; there are experimental data and phenomenological models to confront with QCD sum-rule predictions. Since the Δ sum rule is especially sensitive to the scalar-scalar four-quark condensate [6,104], one may obtain some additional phenomenological constraints on its density dependence [120].

G. Other Approaches

We have focused our study of nucleons at finite density on sum rules that use the formalism outlined in Ref. [18] to calculate *individually* the scalar and vector self-energies of a nucleon in infinite nuclear matter. (This approach was also discussed in Ref. [13].) However, there are several other approaches to analyzing nucleons at finite density. All of these approaches start with the same correlator we have been studying, but calculate different quantities. Furthermore, in some cases a different OPE and different dispersion relations are used.

The work by Drukarev and Levin pioneered many of the common features of finite-density sum rules, such as the linear treatment of the quark and gluon condensates [12]. Their approach is presented in detail in a review article [17] and several journal articles [12,16] and most recently by Drukarev and Ruskin [26]. Here we simply comment on differences between their approach and ours.

One difference is in the choice of external kinematics, which leads to a different OPE. Rather than exploiting the similarities to the vacuum sum rules, as we have done by choosing unphysical kinematics that imply a short-distance expansion, Drukarev and Levin advocate an analogy to deep-inelastic scattering. In this approach, the spacelike limit $q^2 \to -\infty$ is taken with $q^2/q\cdot u$ fixed, which is similar to the Bjorken limit. This ensures that only the light cone is probed, but does not imply a short-distance expansion. Thus the appropriate OPE is a light-cone expansion, and one must deal with all operators of a given twist.

These authors also consider finite-density dispersion relations in q^2 (rather than q_0), with $s = (p+q)^2$ fixed (instead of fixing $\mathbf{q}$). Here p^μ is the four-momentum of a nucleon in the medium [17]. These dispersion relations are assumed, rather than constructed through a Lehmann representation. One difficulty with this approach is that the identification of quasinucleon intermediate states becomes obscured.

Drukarev and Levin use their finite-density sum rules to describe nuclear matter saturation properties. (They have also calculated g_A in the medium [17].) Thus, rather than focusing on individual scalar and vector self-energies, they study the shift of the quasinucleon pole. Cancellations between scalar and vector contributions are still present in the sum rules and were pointed out in Ref. [17]. Since the empirical pole shift is quite small on hadronic scales, however, its determination from sum rules is likely to be very uncertain. For example, one would require detailed knowledge of the density dependence of the condensates; it is clear that our present understanding of this density dependence is insufficient. We argue that one can establish whether QCD predicts large and canceling scalar and vector self-energies far more reliably than one can quantitatively predict the net single-particle energy.

The work by Henley and Pasupathy [23] is based heavily on the development of Drukarev and Levin, but with a focus on different observables. Instead of nuclear-matter saturation properties, Henley and Pasupathy calculate nucleon-nucleus scattering. In particular, they expand the correlator to linear order in the density, which allows them to isolate the forward nucleon-nucleus scattering amplitude from the residue of the double pole at the nucleon mass. The advantage to this calculation is that they make a more direct connection to actual physical observables, the scattering amplitudes. However, as noted in the Introduction, QCD sum rules are not well suited for calculating such quantities because of delicate cancellations.

In a recent Letter [121], Kondo and Morimatsu advocate the calculation of nucleon-nucleon scattering lengths using QCD sum-rule methods. At the same time, they argue that the calculations we have described in this review are ill-founded. We disagree strongly with both aspects of their discussion. The essential criticism is given in Refs. [80,122]; here we just summarize the main points.

The first problem is that the QCD sum-rule approach is not appropriate for the calculation of NN scattering lengths. The large magnitudes of the empirical NN scattering lengths reflect the slightly and nearly bound states in the two scattering channels. It is well known from conventional nuclear physics that making very small changes in the strength of an assumed NN potential can make calculated scattering lengths arbitrarily large or change their signs. Sum rules cannot hope to address the fine details of this physics, which is determined at relatively large distances or times.

Despite this, the sum-rule calculations of Kondo and Morimatsu appear to predict scattering lengths qualitatively similar to the experimental data. However, their sum rules suffer from a subtle flaw that has been alluded to earlier in this review. Their predictions

are acutely sensitive to asymmetries in the spectral density between positive- and negative-energy states. They neglected to account for this asymmetry in the effective continuum thresholds in their analysis, and this led to spurious results (see Ref. [80] for details). If one accounts properly for these asymmetries in their approach, one finds results consistent with ours [80].

IV. HYPERONS IN NUCLEAR MEDIUM

A. Overview

Large and canceling Lorentz scalar and vector self-energies for the in-medium nucleon, which are predicted by relativistic phenomenology, are made plausible by the nucleon sum-rule results. Yet, as we have stressed, these results are also inconclusive at present; thus it is important to test the sum-rule framework against finite-density phenomenology and experiment in other contexts. In this section, we discuss finite-density QCD sum rules for the self-energies of Λ and Σ hyperons in nuclear matter.

In hypernuclei, hyperons can occupy the lowest (innermost) shell-model orbits in a nucleus, which should allow reasonable comparisons to theoretical predictions for hyperons in uniform nuclear matter. Furthermore, various investigators have extended relativistic phenomenology to the study of hypernuclear physics [123–133]. In these relativistic models, the hyperon propagating in the nuclear medium is described by a Dirac equation with scalar and vector potentials. The potential depths result from the coupling of the hyperons to the same scalar and vector fields as the nucleon, but with different coupling strengths; however, these coupling strengths are not well determined. By solving the Dirac equation, one can obtain the binding energies and spin-orbit splittings for different states in hypernuclei, which can be confronted with experimental data.

One of the compelling successes of relativistic models in describing nucleon-nucleus interactions is the naturally large spin-orbit force for nucleons in a finite nucleus. This force depends on the derivatives of the scalar and vector optical potentials, which add constructively. An analogous prediction for the Λ hyperon would seem to be problematic, if one adopts the *naive* quark-model prediction that each coupling for the Λ should be 2/3 the coupling for the nucleon [134,135,126–130]. [That is, if one assumes that the scalar (σ) and vector (ω) mesons couple exclusively to the up and down (constituent) quarks and not to the strange quark.] In the Λ-nucleus system, recent experiments indicate that the spin-orbit force is small, and even consistent with zero [136,137]. This has raised questions about the validity of relativistic nuclear phenomenology for hyperons.

An early work [123] achieved a small spin-orbit force by taking the potentials (*i.e.*, the couplings) for the Λ to be a factor of three smaller than for the nucleon. More recently, however, it has been suggested that scalar and vector coupling strengths consistent with the naive quark-model predictions can be used if a tensor coupling between hyperons and the vector meson (ω) [126–130], motivated from a quark-model picture [126,128,129], is introduced. In this picture, the tensor couplings of the hyperons (Λ, Σ, Ξ) to the vector meson differ in their magnitudes and signs, and hence their contributions to the spin-orbit force are different. (This picture yields a negligible tensor coupling for the nucleon.) For

the Λ, this picture leads to a tensor coupling with strength equal in magnitude to the corresponding vector coupling, but with the opposite sign. The net result, in combination with the scalar contribution, is a small spin-orbit force. A recent global fit to hypernuclear data favors smaller couplings (from 0.3 to 0.5 times the nucleon-meson couplings, depending on details [132]), but does not include this new tensor coupling.

While experimental evidence of Σ hypernuclei is lacking at present, a number of authors have extended the relativistic phenomenology to Σ hypernuclei and presented theoretical predictions [128–131]. In Refs. [128–130], the naive quark-model prediction for the coupling strengths of the Σ to the scalar and vector mesons was adopted (the vector coupling for the Σ is 2/3 the coupling for the nucleon, and the scalar coupling for the Σ is slightly smaller than 2/3 the coupling for the nucleon); in addition, the tensor coupling between the Σ and the vector meson was included. In the quark-model picture of Ref. [128], the tensor coupling of the Σ to ω has the *same* sign as the corresponding vector coupling, in contrast to the Λ case. With the quark-model values for the tensor coupling, the spin-orbit force for the Σ was found to be comparable with the nucleon spin-orbit force. In Ref. [131], the tensor coupling was omitted and universal couplings were assumed for *all* hyperons; the ratio of the Σ to Λ spin-orbit force obtained is about 0.9.

QCD sum-rule predictions for the scalar and vector self-energies of hyperons may offer independent information on the scalar and vector couplings (or potential depths) for hyperons in the nuclear medium. In addition, they may offer new insight into possible SU(3) symmetry-breaking effects in the baryon self-energies and into the spin-orbit forces for the hyperons. We note, however, that the prediction of tensor couplings is not tested in the sum-rule results described below.

The finite-density sum rules for hyperons can be obtained following the same methods as discussed in the previous sections of this review. Here we will omit the details of the derivations and the actual sum rules, and focus on the predictions for hyperons. The reader is referred to Refs. [22,27] for further details.

B. Λ Hyperons

The in-medium correlator for the Λ is defined by

$$\Pi_\Lambda(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle \Psi_0 | T\eta_\Lambda(x)\overline{\eta}_\Lambda(0)|\Psi_0\rangle \ , \tag{4.1}$$

where η_Λ is a color-singlet interpolating field with the spin, isospin, and strangeness of the Λ. As in the nucleon case, we consider interpolating fields that contain no derivatives and couple to spin-$\frac{1}{2}$ only. Here we choose the interpolating field

$$\eta_\Lambda = \sqrt{\frac{2}{3}}\epsilon_{abc}[(u_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu d_c - (d_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c] \ , \tag{4.2}$$

which has been used in the studies of vacuum sum rules for the Λ [138,8,139]. In the SU(3) limit, this interpolating field leads to the same results as Ioffe's current for the nucleon.

Since the Λ hyperon decays only weakly, its width in free space can be neglected on hadronic scales. Experimentally, bound Λ single-particle states with well-defined energies

have been observed in Λ hypernuclei, which indicates that a quasiparticle description of the Λ is reasonable; thus, one can adopt a pole approximation for the quasilambda. As in the nucleon sum-rule analysis, we choose a weighting function to suppress the contribution from negative-energy excitations (corresponding to the antiparticle).

The operator product expansion for the Λ correlator is a direct generalization of the nucleon OPE and is given in detail in Ref. [22]. The new features are the contributions of condensates involving the strange quark (such as $\langle\overline{s}s\rangle_{\rho_N}$, see Sec. III E 2) and the strange-quark mass m_s. There are two dimension-six scalar-scalar four-quark condensates in the Λ sum rules: $\langle\overline{q}q\rangle^2_{\rho_N}$ and $\langle\overline{q}q\rangle_{\rho_N}\langle\overline{s}s\rangle_{\rho_N}$ [22]. As stressed in the nucleon case, the in-medium factorization approximation may not be justified in nuclear matter. Thus, we follow the parametrization in Eq. (3.134) for the density dependence of $\langle\overline{q}q\rangle^2_{\rho_N}$ and parametrize the density dependence of $\langle\overline{q}q\rangle_{\rho_N}\langle\overline{s}s\rangle_{\rho_N}$ as

$$\widetilde{\langle\overline{q}q\rangle_{\rho_N}\langle\overline{s}s\rangle_{\rho_N}} = (1-f')\langle\overline{q}q\rangle_{\rm vac}\langle\overline{s}s\rangle_{\rm vac} + f'\langle\overline{q}q\rangle_{\rho_N}\langle\overline{s}s\rangle_{\rho_N} \ , \tag{4.3}$$

where f' is a real parameter. The f' values are taken to be in the range $0 \leq f' \leq 1$ such that the condensate interpolates between its factorized form in free space and its factorized form in the nuclear medium.

In Fig. 7, the optimized results for the normalized self-energies M^*_Λ/M_Λ and Σ_v/M_Λ are displayed as functions of f' for $m_s = 150\,\mathrm{MeV}$, $y = 0.1$, $|\mathbf{q}| = 270\,\mathrm{MeV}$, and three different values of f. One notes that Σ_v/M_Λ is insensitive to both f and f'. M^*_Λ/M_Λ, however, varies rapidly with f and f'; therefore, the sum-rule prediction for the scalar self-energy is *strongly* dependent on the density dependence of the four-quark condensates. For $f = 0.25$ and values of f' in the range $0.6 \leq f' \leq 1$, the predictions are $M^*_\Lambda/M_\Lambda \simeq 0.85$–$0.94$ and $\Sigma_v/M_\Lambda \simeq 0.09$. On the other hand, for $f = 0.25$ and small values of f' ($0 \leq f' \leq 0.3$), one finds $\Sigma_v/M_\Lambda \simeq 0.08$ and $M^*_\Lambda/M_\Lambda \simeq 0.68$–$0.76$. As f increases, M^*_Λ/M_Λ decreases, which implies an even larger magnitude for the scalar self-energy. (In the nucleon case, M^*_N/M_N *increases* as f increases.) Thus, a weak density dependence of *both* scalar-scalar four-quark condensates leads to a moderate vector self-energy and a very strong scalar self-energy (the net self-energy is strong and attractive). (A strong density dependence for $\langle\overline{q}q\rangle^2_{\rho_N}$ also yields a strong scalar self-energy.) The sum-rule predictions for the self-energies turn out to be insensitive to the values of m_s, y, and $|\mathbf{q}|$ [22].

FIG. 7. Optimized sum-rule predictions for M_Λ^*/M_Λ and Σ_v/M_Λ as functions of f'. The three curves correspond to $f=0$ (solid), $f=0.25$ (dashed), and $f=0.5$ (dotted).

We observe that the prediction for the normalized vector self-energy Σ_v/M_Λ is insensitive to the details of the calculations. For typical values of the relevant condensates and other input parameters, $\Sigma_v/M_\Lambda \simeq 0.08$–$0.09$. The finite-density *nucleon* sum rules predict $\Sigma_v/M_N \simeq 0.25$–0.30. Thus, one finds $(\Sigma_v)_\Lambda/(\Sigma_v)_N \simeq 0.3$–$0.4$. This result, if interpreted in terms of a relativistic hadronic model, would imply that the coupling of the Λ to the Lorentz vector field is weaker than the corresponding nucleon coupling by the same ratio. This compares to the naive quark-model prediction of 2/3, which is obtained by assuming that the mesons couple directly to constituent quarks.

The predictions for the scalar self-energy are quite sensitive to the undetermined density dependence of certain four-quark condensates. If one assumes that the four-quark condensate $\langle\overline{q}q\rangle_{\rho_N}^2$ depends only weakly on the nucleon density (*i.e.*, if f is small) and the four-quark condensate $\langle\overline{q}q\rangle_{\rho_N}\langle\overline{s}s\rangle_{\rho_N}$ has a strong density dependence (*i.e.*, if f' is large), one finds $M_\Lambda^*/M_\Lambda \simeq 0.85$–$0.94$, which implies $\Sigma_s/M_\Lambda \simeq -(0.06$–$0.15)$. With the nucleon sum-rule prediction, $M_N^*/M_N \simeq 0.65$–0.70, one obtains $(\Sigma_s)_\Lambda/(\Sigma_s)_N \simeq 0.2$–$0.4$. In a hadronic model, this implies again a much smaller coupling of the Λ to the Lorentz scalar field than for the nucleon. In this case, there is a significant degree of cancellation between the scalar and vector self-energies. This result is compatible with the empirical observation that the Λ single-particle energy (the quasiparticle pole position) is shifted only slightly in nuclear matter relative to its free space mass. That is, the Λ is very weakly bound.

On the other hand, if *both* $\langle\overline{q}q\rangle_{\rho_N}^2$ and $\langle\overline{q}q\rangle_{\rho_N}\langle\overline{s}s\rangle_{\rho_N}$ depend either weakly or moderately on the nucleon density, the predicted ratio M_Λ^*/M_Λ is significantly smaller than unity, implying that the scalar self-energy is large and negative. The predicted vector self-energy, on the other hand, is still moderate. Thus, in this case the sum rules predict incomplete cancellation and the resulting quasilambda energy is significantly smaller than the mass. This result is inconsistent with experiment.

C. Σ Hyperons

The sum rules for Σ hyperons are obtained by studying the correlator

$$\Pi_\Sigma(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle \Psi_0 | T\eta_\Sigma(x)\overline{\eta}_\Sigma(0) | \Psi_0 \rangle \ , \tag{4.4}$$

where η_Σ is an interpolating field with the quantum numbers of a Σ. The interpolating field for the Σ^+ can be obtained directly by an SU(3) transformation of Ioffe's current for the nucleon [138,8]:

$$\eta_{\Sigma^+} = \epsilon_{abc}(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu s_c \ . \tag{4.5}$$

The analogous interpolating field for Σ^0 follows by changing one of the up-quark fields into a down-quark field and then symmetrizing the up and down quarks, which can also be written as:

$$\eta_{\Sigma^0} = \sqrt{2}\epsilon_{abc}[(u_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu d_c + (d_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c] \ . \tag{4.6}$$

(Note that η_{Σ^0} and η_Λ in Eq. (4.2) have an identical structure, except for the isospin couplings of the up- and down-quark fields.) The results obtained by using η_{Σ^+} and η_{Σ^0} are the same due to isospin symmetry.

The width of the Σ in free space is small and can be ignored on hadronic scales. At finite density, the width of the Σ will be broadened due to strong conversions; here we assume that the broadened width is relatively small on hadronic scales and that a quasiparticle description of the Σ is reasonable. In the context of relativistic phenomenology, the Σ is assumed to couple to the same scalar and vector fields as a nucleon in nuclear matter and is treated as a quasiparticle with real Lorentz scalar and vector self-energies. Thus a pole ansatz is adopted in our study, and Eq. (3.110) can be utilized to minimize the sensitivity to the negative-energy excitations.

The expressions for Π_Σ and the three finite-density sum rules are given in Ref. [27]. Note that only the scalar-scalar four-quark condensate $\langle\overline{q}q\rangle^2_{\rho_N}$ appears in the Σ sum rules. We use the parametrization of Eq. (3.134) for its density dependence.

The optimized sum-rule predictions for the ratios M_Σ^*/M_Σ and Σ_v/M_Σ are plotted as functions of y in Fig. 8 for $m_s = 150\,\text{MeV}$, $|\mathbf{q}| = 270\,\text{MeV}$, and three different values of f. Again, the ratio Σ_v/M_Σ is insensitive to both y and f. The ratio M_Σ^*/M_Σ, on the other hand, changes rapidly with y and f, which implies that the sum-rule prediction for the scalar self-energy is *strongly* dependent on the strangeness content of the nucleon and on the density dependence of the four-quark condensate. For $f = 0$ and values of y in the range $0.4 \leq y \leq 0.6$, the predictions are $M_\Sigma^*/M_\Sigma \simeq 0.78$–$0.85$ and $\Sigma_v/M_\Sigma \simeq 0.18$–$0.19$. On the other hand, for $f = 0$ and small values of y ($0 \leq y \leq 0.2$), one gets $\Sigma_v/M_\Sigma \simeq 0.18$ and $M_\Sigma^*/M_\Sigma \simeq 0.92$–$0.98$. As f increases, M_Σ^*/M_Σ increases, which implies an even smaller magnitude of the scalar self-energy. The sum-rule predictions are insensitive to the values of m_s and $|\mathbf{q}|$ [27].

FIG. 8. Optimized sum-rule predictions for M_Σ^*/M_Σ and Σ_v/M_Σ as functions of y. The three curves correspond to $f = 0$ (solid), $f = 0.25$ (dashed), and $f = 0.5$ (dotted).

The sum-rule prediction for the normalized vector self-energy Σ_v/M_Σ is apparently insensitive to the details of the calculations. For typical values of the relevant condensates and other input parameters, one obtains $\Sigma_v/M_\Sigma \simeq 0.18$–$0.21$. The finite-density *nucleon* sum rules predict $\Sigma_v/M_N \simeq 0.25$–0.30; thus, we find $(\Sigma_v)_\Sigma/(\Sigma_v)_N \simeq 0.8$–$1.1$. This result, if interpreted in terms of a relativistic hadronic model, would imply that the coupling of the Σ to the Lorentz vector field is very similar to the corresponding nucleon coupling. This compares to the naive quark-model prediction of 2/3, which is obtained by assuming that the mesons couple directly to constituent quarks [134,135,126–130].

The sum-rule prediction for the scalar self-energy is sensitive to the strangeness content of the nucleon and the density dependence of the four-quark condensate $\langle \overline{q}q\rangle^2_{\rho_N}$. If we assume that the nucleon has a large strangeness content (*i.e.*, $y \simeq 0.4$–0.6) and the four-quark condensate $\langle \overline{q}q\rangle^2_{\rho_N}$ depends only weakly on the nucleon density (*i.e*, if $f \simeq 0$), we find $M_\Sigma^*/M_\Sigma \simeq 0.77$–$0.84$, which implies $\Sigma_s/M_\Sigma \simeq -(0.16$–$0.23)$. With the nucleon sum-rule prediction $M_N^*/M_N \simeq 0.65$ (for $f \simeq 0$), we obtain $(\Sigma_s)_\Sigma/(\Sigma_s)_N \simeq 0.6$–$0.85$. In a hadronic model, this implies again a coupling of the Σ to the Lorentz scalar field close to that for the nucleon. In this case, there is a significant degree of cancellation between the scalar and vector self-energies, which is compatible with that implemented in the relativistic phenomenological models. [We note, however, that Eq. (3.94) suggests values of the strangeness content in the range $y \simeq 0$–0.45.]

In contrast, if the strangeness content of the nucleon is small (*i.e.*, $y \leq 0.2$) or if $\langle \overline{q}q\rangle^2_{\rho_N}$ has a significant dependence on the nucleon density, the predicted ratio M_Σ^*/M_Σ is close to unity, implying that the scalar self-energy is very small. The predicted vector self-energy, on the other hand, is still essentially the same as the nucleon vector self-energy. Thus, in this case, the sum rules predict incomplete cancellation and, hence, a sizable repulsive net self-energy for the Σ. This result is at odds with the predictions of relativistic models.

D. Summary

The QCD sum rules discussed here predict that the Λ vector self-energy is significantly smaller than the nucleon vector self-energy while the Σ vector self-energy is close to that for the nucleon. However, just as for the nucleon, the sum-rule predictions for the hyperon scalar self-energies are obscured by the strong dependence on the values of four-quark condensates and on the strangeness content of the nucleon (in the Σ case).

Despite the uncertainties, one finds in general that sum-rule predictions for the scalar and vector self-energies imply a much weaker spin-orbit force for the Λ in a nucleus than that felt by a nucleon. The predictions for the Σ scalar and vector self-energies seem to imply a spin-orbit force for the Σ comparable to that for a nucleon, but much stronger than that for a Λ. These results are compatible with experiment and with predictions of relativistic phenomenological models with an extra tensor coupling between the hyperons and the vector meson [126–130]. Note that the magnitude of the tensor contribution cannot be estimated from QCD sum rules in uniform nuclear matter.

It is worth emphasizing that in Refs. [126–130], scalar and vector couplings consistent with the naive quark-model predictions have been adopted, and it is the extra tensor coupling of the hyperons to the vector meson that reduces the Λ spin-orbit force and enhances the Σ spin-orbit force. The sum-rule predictions, on the other hand, suggest that it is the weak scalar and vector couplings for the Λ and strong scalar and vector couplings for the Σ, deviating from the naive quark-model prediction, that lead to a small Λ spin-orbit force and a large Σ spin-orbit force, respectively. We also note that the sum-rule predictions do not agree with the universal coupling assumption (*i.e.*, all hyperons couple to the scalar and vector fields with the same strength) suggested in Ref. [131].

V. VECTOR MESONS IN MEDIUM

A. Introduction

This review has primarily stressed the problem of baryons—nucleons and hyperons—propagating in nuclear matter. In this section, we will briefly consider a different problem, namely the propagation of vector mesons in the nuclear medium. Much of the sum-rule machinery carries over directly from our previous discussion; further details can be found in the cited literature.

The possibility that the properties of vector mesons might change significantly with increasing density is of considerable current theoretical interest. For example, Brown and Rho [140] have proposed a scenario in which the vector-meson masses in the medium decrease. Experimentally, the question of how vector mesons behave at finite density is an open question. If the masses decrease, then one effect should be an increased range of propagation, so that the effective size of the nucleons as "seen" in hadron reactions mediated by vector mesons will increase. Unfortunately, it is difficult to isolate the meson-mass effect of "swollen" nucleons due to decreased vector-meson masses from other physics arising from nuclear many-body effects or the substructure of the hadrons. Given this generic difficulty, attempts to extract the meson-mass effect are fraught with ambiguity.

Various pieces of experimental evidence have been proposed in support of the picture of decreasing meson masses. One example is the quenching of the longitudinal response (relative to the transverse response) in quasielastic electron scattering [141] and $(e, e'p)$ [142] reactions, which might reflect "swollen" nucleons [143,144] in the medium. As noted in Refs. [52,53], this swollen-nucleon picture emerges naturally if the electromagnetic current couples to the nucleons through the vector meson (at least partly) and if the vector-meson mass in medium drops. A second example is the discrepancy between the total cross section in K^+-nucleus scattering on ^{12}C and that predicted from an impulse approximation calculation using K^+-nucleon scattering amplitudes (extracted from K^+-D elastic scattering). The discrepancy is removed [54] if there is an effective increase in the nucleon's in-medium cross section due to a decrease in the mass of the ρ meson mediating the interaction.

More direct investigations of vector-meson masses in the nuclear medium have also been proposed. One proposal is to study dileptons as a probe of vector mesons in the dense and hot matter formed during heavy-ion collisions [55]. The dilepton mass spectra should allow one to reconstruct the mass of vector mesons decaying electromagnetically. A potentially cleaner probe (less leptonic background) would be via virtual compton scattering—the $(\gamma, e^- e^+)$ reaction [56].

There are a number of potential difficulties with these probes. One concerns the lifetimes of the vector mesons. The ω and ϕ mesons are rather long-lived; thus, there is a large probability that, regardless of where they are created, they will decay *outside* the region in which the matter is dense. This raises the question of whether these decays will provide useful information about the dense matter region. A second difficulty is that, in the nuclear medium, the position of the pole (assuming pole dominance) is not a function of the four-momentum q^2 only; it can also depend explicitly on the three-momentum $\mathbf{q}$. Thus, studies of the invariant mass of the dileptons may not be sufficient to pick out cleanly the effects of mass shifts. Finally, there is no reason to suppose that the longitudinal and transverse components behave in the same way—indeed one expects them to be different, and their difference may complicate attempts to extract the masses from data.

B. Formulation of the Sum Rule

The study of vector mesons in the nuclear medium via QCD sum rules offers a theoretical complement to these experimental efforts. The pioneering studies at finite density were by Hatsuda and Lee [19], and there have been subsequent calculations by Asakawa and Ko [24]. The treatment is quite similar to the treatment of the baryons discussed in previous sections of this review. Here we will only sketch the analysis, with an emphasis on the differences between the baryon and vector-meson cases. Our notation will not follow the notation of Refs. [19,24] precisely; instead we introduce a more general form for the correlator that allows for calculations away from the $\mathbf{q} = 0$ point.

The interpolating fields used for these studies and for the studies of vector mesons in the vacuum [5] are the conserved vector currents of QCD:

$$J^{\rho}_{\mu} = \tfrac{1}{2}(\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d) \ , \qquad J^{\omega}_{\mu} = \tfrac{1}{2}(\overline{u}\gamma_\mu u + \overline{d}\gamma_\mu d) \ , \qquad J^{\phi}_{\mu} = \overline{s}\gamma_\mu s \ . \tag{5.1}$$

The correlator is defined as

$$\Pi_{\mu\nu}(q) \equiv \int d^4x\, e^{iq\cdot x} \langle \Psi_0 | T J_\mu(x) J_\nu(0) | \Psi_0 \rangle \ , \tag{5.2}$$

where $|\Psi_0\rangle$ is the nuclear matter ground state, T is the covariant time-ordering operator [61], and J_μ represents any of the three vector currents.

To proceed, it is necessary to identify the various tensor structures that can appear in this problem. A simplification is that each of these currents is exactly conserved: $\partial_\mu J^\mu = 0$. The covariant time-ordered product is defined such that [61]

$$q^\mu \Pi_{\mu\nu}(q) = q^\nu \Pi_{\mu\nu}(q) = 0 \ . \tag{5.3}$$

This constraint plus the fact that there are two four-vectors, q_μ and u_μ, from which to construct tensor structures allows us to deduce that there are two independent tensor structures:

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_T(q^2, (q\cdot u)^2) + q^2 \left(\frac{n_\mu n_\nu}{n^2} \right) [\Pi_T(q^2, q\cdot u) - \Pi_L(q^2, (q\cdot u)^2)] \ , \tag{5.4}$$

where we define $n_\mu \equiv u_\mu - (q\cdot u/q^2) q_\mu$. It is easy to see that each of these structures satisfies Eq. (5.3). The first term in Eq. (5.4) is the familiar structure seen in the vacuum correlator; clearly, the second term must vanish as the density goes to zero. We choose to normalize the longitudinal and transverse correlators, Π_L and Π_T, such that, with $q^\mu = (\omega, 0, 0, |\mathbf{q}|)$, $\Pi_{00} = \mathbf{q}^2 \Pi_L$, $\Pi_{11} = \Pi_{22} = q^2 \Pi_T$, and $\Pi_{33} = \omega^2 \Pi_L$.

In principle, one can derive sum rules for Π_L and Π_T, or any linear combination of the two. In practice, all calculations done to date have been with $\mathbf{q} = 0$ (in the rest frame of the matter); as $\mathbf{q} \to 0$ the longitudinal and transverse polarizations become degenerate, since there is no way to distinguish longitudinal from transverse. The issue of whether sum rules for certain combinations of invariant functions are more effective than others will arise, however, as soon as calculations away from $\mathbf{q} = 0$ are attempted.

In the baryon case, we noted that the correlation function, considered in the rest frame as a function of q_0, has both even and odd parts. The reason is that, at finite baryon density, a baryon in medium propagates differently than an antibaryon, yielding a correlation function that is asymmetric in the energy variable. In contrast, for the correlators in the vector channels, both time orderings correspond to the creation of the vector meson, which is its own antiparticle. Accordingly, the spectral function is necessarily an even function of energy, and one can write the dispersion relation as an integral over the energy squared.[18] Since we work here at $\mathbf{q} = 0$ in the rest frame of the medium, we can write the correlator as $\Pi_L(\omega^2)$ with $\omega \equiv q_0$.

Having decided to fix $\mathbf{q} = 0$, the calculation goes forward in the standard way. One starts with a dispersion relation

$$\Pi_L(\omega^2) = \frac{1}{\pi} \int_0^\infty ds \, \frac{\mathrm{Im}\, \Pi_L(s)}{s - \omega^2 - i\epsilon} \ . \tag{5.5}$$

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[18] This is true for *neutral* vector mesons in any type of baryonic matter. For charged ρ mesons, however, ρ^+ is the antiparticle of ρ^-. In an isoscalar medium, they are degenerate and the spectral function is even. If, however, one considers a medium with a net isospin density (such as neutron matter), they cease to be degenerate and the spectral function is no longer even.

In general we would need to make a subtraction, but as usual we convert to a more effective weighting function in the integral by applying a Borel transform. The result is:

$$\widehat{\Pi}_L(M^2) = \frac{1}{\pi}\int_0^\infty ds\, W(s)\, \mathrm{Im}\,\Pi_L(s)\ , \quad W(s) = e^{-s/M^2} \tag{5.6}$$

In the nucleon case, we had to isolate the nucleon contribution from the antinucleon contribution, which led us to an asymmetric weighting function designed to minimize this contamination (see Sec. III F 1). In the present case, this issue does not arise and we can use the Borel weighting identical to the vacuum case.

To proceed, one follows the same steps described for baryon sum rules:

1. compute the Wilson coefficients;
2. estimate the corresponding condensates;
3. devise a phenomenological model for the spectral density that contains a few free parameters;
4. determine the parameters by finding the best match of the phenomenology to the OPE.

We focus here on some issues raised in these sum rules and refer the reader to the literature [19,24] for specific details of the actual calculations and results (including results for the ϕ meson).

The calculation of the Wilson coefficients involves no new subtleties; the coefficients can be calculated using the techniques outlined in Sec. III D 5. Since finite-density condensates are properties of the medium and not the probe, the same condensates estimated earlier for baryon sum rules enter here. The reader should be reminded that these estimates are rather crude. Typically we only estimate these condensates by folding nuclear matrix elements with the nuclear density. While this may be adequate for normal nuclear matter density, it is likely to be problematic at high densities (*e.g.*, 2–3 times normal nuclear density). This should be kept in mind in the context of heavy-ion collisions, which produce such high densities. The appropriate nuclear matrix elements come from a variety of sources, and, in some cases, modeling is necessary.

In the case of the nucleon, we found that the sum-rule predictions were extremely sensitive to the value of certain dimension-six four-quark condensates. The hypothesis of ground-state saturation led to results in strong contradiction with experiment; furthermore, the theoretical prejudices in favor of the hypothesis are not strongly founded. This issue is also extremely important in the context of the vector-meson sum rules. In the vacuum sum rule for the vector mesons, four-quark condensates provide the dominant correction to the perturbative term in the OPE. (Note, however, that these are not the same four-quark condensates as in the nucleon case.) In the finite-density sum rules, the density dependences of these condensates largely determine the predicted density dependence of the vector-meson mass. Since all in-medium vector-meson calculations to date assume factorization, we must interpret the results with caution.

To implement the sum rule, one needs a model for the spectral function. In Ref. [19], the following ansatz is adopted:

$$R\,\mathrm{Im}\Pi_L(\omega^2) = \rho_{\mathrm{sc}}\delta(\omega^2) + F\delta(\omega^2 - m^{*2}) + \left(1 + \frac{\alpha_s}{\pi}\right)\theta(\omega^2 - s_0) \ , \tag{5.7}$$

with $R = 8\pi$ for the ρ and ω mesons, and $R = 4\pi$ for the ϕ meson. The effective mass m^* corresponds to the position of the propagator pole. The second and third terms of Eq. (5.7) represent a pole-plus-continuum ansatz, exactly as in the vacuum sum rules.

The first term is a scattering term, which corresponds to Landau damping. The form of this scattering term requires some explanation. For a nonzero three-momentum $\mathbf{q}$, the imaginary part of the correlator has support in the *spacelike* region ($\omega^2 < \mathbf{q}^2$). Indeed, in electron scattering these spacelike excitations are all that one can probe. For the propagation of modes, these spacelike excitations act as damping terms. In general, the analytic structure of the correlator in the ω plane corresponding to these spacelike excitations is a cut. However, as $\mathbf{q} \to 0$, this cut gets squeezed to the $q^2 = 0$ point and reduces to a delta function in the imaginary part. A detailed description of the scattering term can be found in Ref. [145]. In practice, the contribution from this term is quite small for the in-medium vector meson sum rules [19].

It is not yet clear to what extent the ansatz in Eq. (5.7) is justified at finite density. A central question is whether there are regions of large spectral density away from the meson pole. One possible source for the ρ-meson channel is from π-Δ-hole states. In model calculations [146], these give rise to significant spectral strength well isolated from the ρ pole. If there is a large low-ω^2 contribution from the π-Δ-hole physics that is not included in the model for the spectral density, the fitted mass for the ρ pole will be significantly lower than the actual mass. The calculation of Asakawa and Ko [24] attempts to include the π-Δ-hole contributions. The results of their sum-rule calculations are quite insensitive to the effects of the π-Δ-hole contributions. On the other hand, the spectral densities extracted by Asakawa and Ko are quite different from those of the model calculations [146].

In the end, Hatsuda and Lee find a substantial reduction in the ρ and ω masses in medium. For low densities, they find [19]

$$\frac{m^*_{\rho,\omega}}{m_{\rho,\omega}} \simeq 1 - (0.18 \pm 0.05)\frac{\rho_N}{\rho_N^{\mathrm{sat}}} \ . \tag{5.8}$$

The decreasing mass arises primarily due to the density dependence of the four-quark condensates and $\langle \overline{q} i D_0 q \rangle_{\rho_N}$. Asakawa and Ko find a somewhat stronger density dependence for the ρ meson, with its mass reduced to about 530 MeV at nuclear matter density. These results are consistent with the hypothesis of decreasing vector meson masses in the nuclear medium, although the reader should keep in mind the many caveats we have cited. In particular, we are concerned about the applicability of a quasiparticle-pole ansatz and the use of factorized four-quark condensates. Future work should consider the density dependence of the coupling constant and the momentum dependence of the longitudinal and transverse effective masses.

VI. DISCUSSION

A. Critical Assessment of the QCD Sum-Rule Approach

The QCD sum-rule approach to hadronic properties is now rather mature. The seminal work of Shifman, Vainshtein, and Zakharov is more than 15 years old and has spawned an entire industry—the original paper [5] has more than 1400 citations. Sum-rule methods have produced phenomenological successes for a wide range of problems, some far beyond the original application of meson masses and couplings. The current state of affairs is summarized in the review/reprint volume edited by Shifman [10].

Despite the popularity and apparent success of QCD sum rules, there remain concerns about the validity of various aspects of the approach. Many of these issues are addressed in Ref. [10]. Here we briefly review some of the questions that carry over in particular to the problem of hadrons in medium.

One central issue is the predictive power of the approach for low-energy hadronic observables. While the OPE may predict with reasonable accuracy certain weighted integrals over physical spectral densities,[19] this is insufficient to make solid predictions for properties of stable particles or resonances. QCD sum rules require nontrivial knowledge of the spectral density, which is used to formulate a spectral ansatz. That is, in order for the sum rules to be predictive, one must already know something about the spectral functions—one cannot make pure *a priori* predictions. This point should be kept in mind when discussing sum rules for hadrons in medium; one's knowledge about the form of spectral densities for this problem is certainly less than for the vacuum case. If assumptions about the finite-density spectral density are qualitatively wrong, then the sum rule may produce highly misleading results. It is accordingly very important to have the best possible model for the phenomenological side.

A second issue at the core of the approach is the validity of the OPE *as actually implemented in QCD sum-rule calculations*. It has been argued [65] that there is *always* an OPE that separates long-distance physics (summarized in condensates) from short-distance physics in Wilson coefficients. The question that must be addressed is the extent to which the Wilson coefficients may be calculated via low-order perturbation theory. Shifman argues [10] that the success of the QCD sum-rule approach phenomenologically justifies the "put all nonperturbative contributions into the condensates and calculate the Wilson coefficients perturbatively" approach for most cases. There are cases in which this approach seems to fail, such as for glueball channels. Others have called it into question for the nucleon channel. For example, in Refs. [72,73], it is claimed that direct-instanton effects in the Wilson coefficients may play an important role in stabilizing the nucleon sum rule. These nonperturbative direct-instanton effects can occur, since instantons may be smaller than the separation scale used in the OPE, and will still play a nonperturbative role. In general the role of instantons in the OPE remains an open question. Our assumption has been that perturbation theory is sufficient to calculate the Wilson coefficients with acceptable accuracy.

A third issue is the estimation of condensates. To the extent that the sum rules are

[19] Assuming that the relevant condensates and Wilson coefficients can be determined with sufficient accuracy.

only sensitive to a few low-dimensional condensates, it is probably reasonable to assume that the important condensates can be determined with sufficient accuracy by fitting to a number of different channels. So long as there are fewer important condensates then there are observables, one will be able to deduce their values from some set of observables and still have predictive power for other observables.

A central question, however, is the validity of the vacuum-saturation or factorization hypothesis for the four-quark condensates, which play an essential role in sum rules for light-quark hadrons. The importance of these condensates despite their relatively high dimension (six) is simply understood—the Wilson coefficients associated with these condensates are anomalously large, because they do not have the small numerical factors associated with loop integrals [5].

In the original work of SVZ [5], a duality argument was given in favor of the vacuum-saturation hypothesis. Subsequently a large-N_c argument was also advanced [111]. In our view, neither argument is especially compelling. The large-N_c argument can be questioned since higher-order terms in the $1/N_c$ expansion are not necessarily negligible given the fact that N_c is only three. The duality argument was based on saturating $\langle\overline{q}q\overline{q}q\rangle$ with both two-pion states and the vacuum:

$$\langle\overline{q}q\overline{q}q\rangle = |\,\langle 0|\overline{q}q|0\rangle\,|^2 + \sum_{2\pi\ \text{states}} |\,\langle 0|\overline{q}q|2\pi\rangle\,|^2\ . \tag{6.1}$$

If one assumes that the two-pion intermediate states are *free* noninteracting pions, then one can show using PCAC-type arguments that the contribution from the pions is much smaller than the vacuum term unless one sums up to very large relative pion energies $\sim 2\,\text{GeV}$. If the sum were cut off by other physics at a more moderate scale (*e.g.*, the ρ-meson mass), then the two-pion contribution is small. However, the two-pion states need not dominate the correction to the vacuum-saturation contribution: multiple pion states can contribute. Moreover, the assumption that the two-pion states behave as free pions is only valid right at threshold.

Apart from these two arguments, the only real justification for the factorization hypothesis is *a posteriori*. Namely, it seems to work phenomenologically in the context of QCD sum rules. For example, Shifman [10] writes that for most channels the phenomenology is consistent with deviations from factorization at only the 10% level or less.

As we have noted, the values of the dimension-six four-quark condensates are the most significant uncertainties in QCD sum-rule treatments of hadrons in nuclear matter. The most naive generalization of vacuum saturation to this problem is ground-state saturation. We can argue based on phenomenological concerns that it is *not* a good approximation at nuclear matter densities. In this context it is worth noting that none of the three arguments in favor of vacuum saturation—duality, large N_c, and phenomenology—give much support to the ground-state saturation hypothesis at finite density. Results given earlier imply that phenomenology is inconsistent with ground-state saturation, at least so far as the nucleon sum rules are concerned.

While neither the duality nor large-N_c arguments are compelling for the vacuum case, for the case of finite density matter they are even weaker. The duality argument was based on the contribution of two-pion states. At finite densities, however, in addition to two-pion states there are very many low-lying particle-hole type states in the system. Thus, it is by

no means clear that the ground state will dominate over contributions from these new states. With regard to the large-N_c argument, it should be noted that the $1/N_c$ expansion alone is known to be unreliable for estimating the relative sizes of nuclear matter observables. For example, a large-N_c analysis gives the kinetic energy of nucleons in nuclear matter as being $O(N_c^{-1})$, while the potential energy is $O(N_c)$. In realistic nuclei at nuclear matter densities, the two quantities are, in fact, of similar magnitudes [147].

A final problem to consider is that our results may to some degree be artifacts of the interpolating fields chosen for the analysis. There is always this danger when predictions are not independent of the interpolating fields, even though there are compelling arguments why one choice of field is superior to another. This sort of question is best answered by numerical simulations. Unlike the case of the vacuum, the prospects for useful numerical simulations of nuclear observables is quite unclear. One can hope, however, for some future lattice calculations that will test the basic ideas of scalar and vector physics that we have explored with sum rules and provide some definite insight into the density dependence of the four-quark condensates.

B. Simple Spectral Ansatz

As emphasized above, in order for sum rules to be useful in practice, one must be able to make a simple parameterization of the spectral function. Typically, for vacuum sum rules, one uses a pole plus continuum ansatz, where the continuum is modeled as the perturbative spectral density (plus OPE contributions) starting at a sharp threshold. Clearly, such a parameterization is a major simplification of a complicated spectrum. The important question is whether this simplification leads to significant uncertainties in the application of QCD sum rules.

While there have not been direct tests of the continuum model based on the Borel-transformed correlator, there have been tests of a lattice analog—the correlator as a function of imaginary time. At three-momentum $\mathbf{q} = 0$ (achieved by summing uniformly over spatial directions), the correlator at Euclidean time τ can be written as[20]

$$\Pi(\tau, \mathbf{q} = 0) = \int_0^\infty d\omega \, e^{-\omega\tau} \rho(\omega) \ , \tag{6.2}$$

where ρ is the spectral density. This lattice correlator simply corresponds to a weighted integral of the spectral density with a different weighting function than in the Borel transform. Like the Borel transform, this weighting suppresses the high-energy states and improves convergence of the OPE [148]. (Note, however, that the suppression is greater with Borel weighting.) We also observe natural association of short times with a broad smearing of the spectral function.

Recently, Leinweber has tested the simple spectral ansatz in fits to lattice data of the nucleon correlator [71]. He found that the pole-plus-continuum model did a good job of

[20]Note that if τ is small, only small x^2 is probed, so a short-distance OPE is appropriate in this regime.

describing the correlator over a surprisingly large range of τ, including most of the intermediate range between the perturbative regime and the large-τ regime. Only the latter regime is dominated by the nucleon pole. Moreover, fits to the nucleon mass based on the ansatz in the intermediate-τ range gave remarkable agreement with the nucleon mass extracted from the correlator evaluated at large times. This test supports the pole-plus-continuum ansatz commonly used in sum-rule applications. Some caveats should be made concerning this test [71]: The lattice calculations were done in the quenched approximation, required a sizable extrapolation in quark mass m_q, and were confined to distances somewhat larger than those used in QCD sum-rule fits. We hope similar calculations without these limitations will be made eventually.

While lattice calculations suggest that the simple pole-plus-continuum model works well in (or at least near) the regime of interest, there are concerns on the horizon. Recently it was pointed out [149] that one can use the known behavior of various quantities with changes in the quark mass (in the small quark-mass limit) to test the consistency of the pole-plus-continuum ansatz. When studying the nucleon correlator with the OPE, one finds terms proportional to $\langle\overline{q}q\rangle_{\rm vac}$, which is known from chiral perturbation theory [59] to behave as follows near the chiral limit:

$$\langle\overline{q}q\rangle_{\rm vac} = \langle\overline{q}q\rangle_{\rm vac}^{\chi}\left(1 - \frac{3}{32\pi^2 f_\pi^2} m_\pi^2 \ln\frac{m_\pi^2}{M_0^2} + O(m_\pi^4)\right) , \tag{6.3}$$

where $\langle\overline{q}q\rangle_{\rm vac}^{\chi}$ is the value of the quark condensate in the chiral limit and M_0 is some mass parameter. Thus, the OPE side of the sum rule has a term that behaves like $m_\pi^2 \ln m_\pi^2$. On the other hand, the leading nonanalytic term (in m_q or m_π^2) in the nucleon mass is known to be $-(3g_A^2/32\pi f_\pi^2)m_\pi^3$ [150]; there is no $m_\pi^2 \ln m_\pi^2$ term. To make the phenomenology consistent with the OPE, the continuum must produce a term that goes as $m_\pi^2 \ln m_\pi^2$ to exactly cancel the term in the OPE. Such a contribution to the continuum is not possible in a simple pole-plus-continuum model. On the other hand, it is possible to use soft-pion theorems to show that, if the continuum model includes pions, then the pion contributions automatically reproduce the chiral log seen in the OPE [151]. A rough estimate of the uncertainty in the nucleon mass due to the neglect of pions in the continuum model is 100 MeV [149], the size of the contribution of the $m_\pi^2 \ln m_\pi^2$ term with M_0 chosen to have a "reasonable" value of $\sim 1\,\mathrm{GeV}$.

C. Does Dirac Phenomenology Make Sense?

One of the principal motivations for using QCD sum rules to describe nucleons propagating in nuclear matter is to test the essential qualitative physics underlying Dirac phenomenology. In light of this goal, some comments on the controversy about the validity of the Dirac approach [41–46,51,48] are appropriate. This controversy has been long running and is not considered as settled. However, we argue that the main criticism of the Dirac phenomenology is not germane to the question of interest here: the magnitudes and signs of the scalar and vector parts of the optical potential.

To those who doubt the entire Dirac approach, the question ultimately comes down to whether it makes sense to describe a composite particle such as the nucleon by a Dirac

equation [41,44–46]. Often this is expressed in terms of the role of Z graphs, *i.e.*, scattering from a positive-energy state into a negative-energy state and back. A fundamental difference between a Dirac description and a Schroedinger description is the possibility of these Z graphs. The essence of the criticism of Dirac phenomenology is that the composite nature of the nucleon suppresses Z graphs and, in doing so, suppresses the relativistic effects [41,44,46]. While one cannot yet directly address this question in QCD, it is clear, in the context of toy models [44,46,51,48] and large-N_c QCD [51], that the composite nature of the nucleon does suppress the coupling to nucleon-antinucleon pairs (relative to what one expects with pointlike Dirac nucleons). What these simple models show is the unsurprising fact that a composite nucleon will not behave the same way as a Dirac nucleon. On the basis of this result one may be tempted to throw out Dirac phenomenology, which is, after all, based on the Dirac equation.

However, this observation is completely beside the point. The key question is the nature of nucleon propagation: What are the on-shell self-energies or optical potentials? Suppose we wish to describe proton-nucleus scattering with an optical potential. To do so, one must suppress explicit reference to both the excitations of the nucleus and the internal excitations of the nucleon (which is a necessary consequence of the nucleon's composite structure). The cost of suppressing explicit references to these degrees of freedom is that we get complex and energy-dependent potentials. Now suppose that one wishes to describe the nucleon scattering problem covariantly as well. The result will be a Dirac optical potential. While we do not know how to calculate the optical potential from first principles, we can use general symmetry considerations to deduce its Lorentz structure. We learn that such an optical potential has both scalar and vector pieces. It is ultimately an empirical question whether the scalar and vector potentials are separately large or small.

How can one reconcile large scalar optical potentials (and hence large Z-graph contributions) in the optical-model description with the fact that compositeness suppresses the amplitude to create $\overline{N}N$ pairs? The simple answer is that there is nothing to reconcile; they are completely different things. The Z graphs in the optical potential need not be associated with virtual $\overline{N}N$ pairs. The intuitive connection between Z graphs and virtual $\overline{N}N$ pairs is the hole picture of Dirac. While this is certainly a useful picture for free or weakly interacting point fermions, for strongly interacting or highly composite fermions, there is no necessary connection.

Recently, Wallace, Gross, and Tjon [48] constructed a solvable toy model of a composite comprised of a fermion-boson bound state. In this model, one sees explicitly how, in a complete treatment of the composite in an external field, virtual composite-anticomposite pairs are suppressed (relative to what would be expected from a point Dirac fermion). At the same time, they show how a description of the same physics without explicit reference to compositeness requires large Z-graph contributions at the level of the effective theory. The point here is that the effects of Z graphs in the effective theory (without explicit reference to the internal structure) does *not* correspond to contributions from virtual pairs of composite particles and antiparticles.

To reiterate our perspective on this issue: The essential physics underlying the success of Dirac phenomenology is the large and canceling scalar and vector parts of the optical potential (self-energy). Questions concerning the role of virtual $\overline{N}N$ states or the validity of the Dirac equation for composite particles, while interesting in their own right, are not

relevant in this context.

VII. SUMMARY AND OUTLOOK

The extension of QCD sum-rule techniques to finite density can provide a bridge between nuclear phenomenology and the underlying theory of quantum chromodynamics. However, when making connections to QCD, one must recognize that most familiar nuclear observables are associated with energy scales much smaller than those of hadronic observables. Furthermore, many or most nuclear *experimental* observables are the result of fine cancellations.

QCD sum rules, which extrapolate from short times with limited resolution, cannot be used to resolve small energy differences or to predict physics that is determined at long times or distances. Therefore one should not be too ambitious in applying sum-rule techniques at finite density. Detailed experimental predictions of conventional nuclear observables are more appropriately studied in the context of effective models of QCD. In this class we would include attempts to understand nuclear matter saturation or nucleon-nucleon scattering lengths quantitatively (although *qualitative* insight might be possible). The alternative that we have proposed is to focus on phenomenology that is associated with energy scales more conducive to a sum-rule treatment.

The most solid result to date connected with finite-density QCD sum rules is the behavior of the chiral condensate with density. The model-independent result [Eq. (3.92)], which predicts the linear density dependence of the condensate, appears to be robust, at least up to nuclear matter saturation density. Although there are still sizable uncertainties in the value of the nucleon σ term, it is known well enough to imply a very substantial change (30–40% reduction) in the chiral condensate inside of nuclei. Furthermore, the change with density has been convincingly related to two-pion exchange physics [50], which provides a natural connection to the scalar meson of meson-exchange phenomenology and establishes that the range is sufficiently large that short-range correlations should not cause drastic changes. The latter point is very important; if correlations dominated the physics, then the value of the *average* condensate would not be very relevant.

Given a large change in $\langle\overline{q}q\rangle_{\rho_N}$, one might anticipate compensating changes in the hadronic spectrum. QCD sum rules offer a direct way to make a connection between such QCD ground-state properties and properties of observed states. Here we have used the sum rules to associate $\langle\overline{q}q\rangle_{\rho_N}$ with the change in the scalar self-energy of a quasinucleon. Similarly, changes in the vector self-energy (of opposite sign) are primary associated with the vector density $\langle q^\dagger q\rangle_{\rho_N}$. Assuming the density dependence of the four-quark condensates to be weak, the resulting self-energies are in qualitative agreement with relativistic phenomenology.

A disclaimer that must be made is that in another context, sum rules for nucleons at finite temperature, changes in the quark condensate have been *mistakenly* identified with a shift in the nucleon mass. The critical point is that one must adequately model the phenomenological spectral density; if not, changes that show up as new features in the

spectral density, could be attributed instead to a shift in the mass by the fitting procedure.[21] We know of no analogous problems with our analysis of baryons in nuclear matter. While the spectral ansatz is quite simple and could, in principle, be missing important features, it is consistent with successful nuclear phenomenology.

Many QCD sum-rule results we have cited are rather indefinite (at least at present). However, the nature of the sum-rule approach should be taken into account when accessing the value of its predictions. As in the zero-density case, it is not the quantitative predictions for hadron masses and other properties that are most important. Indeed, lattice calculations will ultimately provide far more precise determinations of the hadronic spectrum. It is the quantitative relations and qualitative insight that are most valuable, and which will persist to challenge any numerical or model predictions.

It has been said (about vacuum sum rules) that QCD sum rules only work well when making "postdictions" rather than predictions. This is true to some extent, although there are also notable exceptions. More precisely, one can say that the reliability of sum-rule applications, particularly when applied to new observables and in new domains, is not guaranteed without some feedback from experiment. The problem is that subtleties can be missed! One should not, at the same time, underestimate the usefulness of postdictions. After all, the data *is* measured experimentally. What we seek is *understanding* of how it fits into a larger picture. This is what the sum rules can provide.

For example, one might ask whether the decomposition into scalar and vector pieces made in relativistic phenomenology is merely an artificial construction. The sum-rule results tell us there is a reason to pay attention to these pieces separately: The physics is different! Specifically, the scalar self-energy is predominantly associated with changes in the scalar condensate, which is in turn related to chiral symmetry restoration. On the other hand, the vector self-energy is largely determined by the vector condensate, or quark density, which is not dynamical.

The ultimate stumbling block to drawing solid quantitative conclusions from the finite-density sum rules is the density dependence of the four-quark condensates. We have repeatedly emphasized the importance of matrix elements of four-quark field operators in QCD sum rules for light-quark hadrons. This is true at zero density, finite density, and finite temperature. In the vacuum, the factorization hypothesis plays an important role in reducing the inputs to the sum rules to a small number of phenomenological parameters. The validity of factorization at zero density seems to be dependent on the particular four-quark condensate and seems quite reasonable in some cases. However, one must remember that a factor of two uncertainty in a dimension-six condensate only propagates as a sixth root of two uncertainty in the predicted mass of a hadron. Thus one only needs factorization to determine the mass scale rather crudely.

In contrast, our concern is with the density dependence of hadronic properties, which requires the density dependence of condensates. Factorization implies a very rapid density

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[21] One should note that even when this happens, it is not so much a failure of the sum-rule approach as of the application of the methods. Thus, with further insight or checks from alternative approaches such as chiral perturbation theory, one can refine and correct the sum-rule calculation.

dependence for four-quark condensates, which has drastic and adverse effects on the nucleon sum-rule predictions. Nevertheless, there are too many uncertainties associated with the finite-density sum rules to insist that our results imply that the four-quark condensates vary slowly with density.

There are two obvious ways to proceed to clarify the situation, which are being pursued at present. The first is to better model the four-quark condensates [152]. The second way is to concentrate on sum rules that do not rely on the four-quark condensates. The key here is to find new, independent sum rules for the nucleon, taking advantage of interpolating fields with both spin-1/2 and spin-3/2 parts. By considering a mixed correlator with Ioffe's current and the spin-3/2–spin-1/2 current, spin-1/2 intermediate states (including the nucleon) are still projected, but one generates additional sum rules for M_N^* that are independent of the problematic four-quark condensates [104,153]. This sum-rule analysis should provide a clean test of the density dependence of M_N^*.

Finally, we comment on sum-rule predictions of changes in hadron properties under the extreme conditions of density that can be reached experimentally in relativistic heavy-ion collisions. The untangling of experimental signatures for such changes is a formidable challenge. The most promising observables appear to be vector-meson masses, which might be measured by monitoring dilepton production. There are several difficulties with the quantitative sum-rule predictions of these masses at present:

- The principal determining factor in the density dependence of masses are certain four-quark condensates. Predictions to date of large changes rely, once again, on a factorization assumption.

- Even accepting factorization, one also needs to extrapolate the density dependence of the quark condensate far beyond ordinary nuclear densities. This extrapolation is very uncertain at this point in time since predictions from model calculations begin to diverge from each other around nuclear matter saturation density.

- An essential ingredient of the sum-rule approach is a simple but complete model of the phenomenological spectral density. There is the definite possibility of missed physics (such as new excitations and widths) in the vector-meson channel that get translated into spurious predictions for mass shifts.

One can hope that the sum-rule picture and approach will be refined as experimental data is accumulated. At present, we must conclude that extrapolations to high density based on QCD sum rules are not quantitatively reliable.

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